Choose the word from the list below that best matches each phrase.

- composition of transformations
- corresponding parts
- image
- rigid motion
- preimage
- translation
- transformation(s)

1. the figure that results from a transformation ___ image ___
2. the original figure in a transformation ___ preimage ___
3. flipping, sliding, or turning a figure ___ transformation ___
4. two or more transformations in combination ___ composition of transformations ___

Use a word from the list above to complete each sentence.

5. This transformation is an example of a ___ translation ___ because the figure slides in one direction, but does not flip, turn, or change size.

6. This translation is an example of a(n) ___ rigid motion ___ because it preserves distance and angle measures.

7. In a translation, the sides or angles of the preimage and image that have the same lengths or angle measures are ___ corresponding parts ___.

Multiple Choice

8. For the transformation shown at the right, triangle $ABC$ is called the ___ D ___
   - A: corresponding part.  
   - B: composition.  
   - C: image.  
   - D: preimage.

9. What type of transformation is shown at the right? ___ H ___
   - F: a flip  
   - G: a reduction  
   - H: a slide  
   - I: a turn

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9-1  Think About a Plan

Translations

Coordinate Geometry  Quadrilateral $PLAT$ has vertices $P(-2, 0)$, $L(-1, 1)$, $A(0, 1)$, and $T(-1, 0)$. $T_{(-2, -3)}(PLAT) = P'L'A'T'$. Show that $PP'$, $LL'$, $AA'$, and $TT'$ are all parallel.

Understanding the Problem

1. How can you use the three sentences of the problem to help you break the problem down into manageable pieces?

   Answers may vary. Sample: The first sentence contains the given information. The second gives the translation rule I can use to obtain the coordinates of the image. The third directs me toward what I need to prove.

Planning the Solution

2. How can you use a diagram to help you solve the problem?

   Plot the given points and the points of the image on a coordinate grid.

3. Use the given translation to find the image points.

   $P'(0, -3)$, $L'(1, -2)$, $A'(2, -2)$, $T'(1, -3)$

4. Graph $PP'$, $LL'$, $AA'$, and $TT'$. What theorems about parallel lines come to mind when you look at the graph?

   If the slopes of two lines are equal, then the lines are parallel.

Getting an Answer

5. Find the slopes of the lines that connect each original point to its image.

   $\frac{-3}{2}$, $\frac{-3}{2}$, $\frac{-3}{2}$, $\frac{-3}{2}$

6. How can you use the slopes to establish that $PP'$, $LL'$, $AA'$, and $TT'$ are all parallel?

   The lines are parallel because the lines all have the same slope.
Tell whether the transformation appears to be a rigid motion. Explain.

1. yes; preserves distance and angle measures

2. no; does not preserve distance

3. yes; preserves distance and angle measures

4. yes; preserves distance and angle measures

In each diagram, the dashed-line figure is an image of the solid-line figure.

(a) Choose an angle or point from the preimage and name its image.
(b) List all pairs of corresponding sides.

5. (a) Sample: $A'$ is image of $A$.
(b) $AB$ and $A'B'$, $BC$ and $B'C'$, $CA$ and $C'A'$

6. (a) Sample: $M$ is image of $V$.
(b) $VW$ and $MJ$, $WX$ and $JK$, $XU$ and $KL$, $UV$ and $LM$

Graph the image of each figure under the given translation.

7. $T_{<-1,4>}(\triangle ABC)$

8. $T_{<3,3>}(MNOP)$

The dashed-line figure is a translation image of the solid-line figure. Write a rule to describe each translation.

9. $T_{<-3,-4>}(ABCD)$

10. $T_{<4,-3>}(UVWXY)$
11. You are visiting Washington, D.C. From the American History Museum you walk 5 blocks east and 1 block south to the Air and Space Museum. Then you walk 8 blocks west to the Washington Monument. Where is the Washington Monument in relation to the American History Museum? 1 block south and 3 blocks west

12. You and some friends go to a book fair where booths are set out in rows. You buy drinks at the refreshment stand and then walk 8 rows north and 2 rows east to the science fiction booth. Then you walk 1 row south and 2 rows west to the children’s book booth. Where is the children’s book booth in relation to the refreshment stand? 7 rows north


14. $\triangle XYZ$ has coordinates $X(2, 3)$, $Y(1, 4)$, and $Z(8, 9)$. A translation maps $X$ to $X’(4, 7)$. What are the coordinates for $Y’$ and $Z’$ for this translation? $Y’(3, 8); Z’(10, 13)$

15. Use the graph at the right. Write three different translation rules for which the image of $\triangle RST$ has a vertex at the origin. $T_{-1, -3}(\triangle RST); T_{-5, -1}(\triangle RST); T_{-2, 2}(\triangle RST)$

16. Use the graph at the right. Write three different translation rules for which the image of $\triangle BCD$ has a vertex at the origin. $T_{3, -3}(\triangle BCD); T_{-4, -1}(\triangle BCD); T_{0, 2}(\triangle BCD)$

Graph the image of each figure under the given translation.

17. $T_{-3, 4}(\triangle DEF)$

18. $T_{-5, 1}(KLMN)$
Tell whether the transformation appears to be a rigid motion. Explain.

1. yes; preserves distance and angle measures

2. no; does not preserve distance and angle measures

In each diagram, the dashed-line figure is an image of the solid-line figure.
(a) Choose an angle or point from the preimage and name its image.
(b) List all pairs of corresponding sides.

3. (a) Answers may vary. Sample: $P$ maps to $P'$; (b) $PQ$ and $P'Q'$; $QR$ and $Q'R'$; $PR$ and $P'R'$

4. $U \rightarrow U'$
   (a) Answers may vary. Sample: $\angle U$ maps to $\angle U'$; (b) $RS$ and $R'S'$; $ST$ and $S'T'$; $TU$ and $T'U'$; $UR$ and $U'R'$

Copy each graph. Graph the image of each figure under the given translation.

5. $T_{<3,-4>}(MATH)$
   Describe in words the translation to the right 3 units and down 4 units.
   To start, identify the coordinates of each vertex.
   The vertices are:
   $M(-4,1), A(-3,3), T(-2,3),$ and $H(-1,1)$.

6. $T_{<-2,3>}($$\triangle ABC$$)$

7. $T_{<-2,-3>}($$\triangle XYZ$$)$
The dashed-line figure is a translation image of the solid-line figure. Write a rule to describe each translation.

8. To start, identify the coordinates of the vertices of both figures.

   The vertices of the preimage are:
   \[ A(-3, 2), B(-3, 4), \text{ and } C(1, 2) \].

   The vertices of the image are:
   \[ A'(1, -4), B'(1, -1), \text{ and } C'(3, -3) \].

   The translation rule is \[ T_{2, -5} \](\( \Delta ABC \)).

9. \[ T_{5, 5}(WXYZ) \]

10. \[ T_{-6, -4}(ABCD) \]

11. You and your friends are visiting a city with blocks laid out in a grid. You walk 7 blocks north and 3 blocks west to a restaurant. After you eat, you then walk 10 blocks east and 3 blocks south to meet up with a friend. Describe your final location based on your starting point.

   4 blocks north and 7 blocks east

12. \( \triangle ABC \) has coordinates \( A(2, 3), B(4, -2), \text{ and } C(3, 0) \). After a translation the coordinates of \( A' \) are \( (6, -1) \). What are the coordinates of \( B' \) and \( C' \)?

   \( B'(8, -6), C'(7, -4) \)

13. Use the graph to the right. Write three different translation rules for which the image of \( \triangle RST \) has a vertex at the origin.

   \[ T_{-3, -2}(\triangle RST); T_{-5, -6}(\triangle RST); T_{-7, -2}(\triangle RST) \]
Multiple Choice

For Exercises 1–4, choose the correct letter.

1. In the diagram, $\triangle A'B'C'$ is an image of $\triangle ABC$. Which rule describes this translation?  
   - $\text{A} \circ T_{-5, -3}(\triangle ABC)$
   - $\text{B} \circ T_{5, 3}(\triangle ABC)$
   - $\text{C} \circ T_{-3, -5}(\triangle ABC)$
   - $\text{D} \circ T_{3, 5}(\triangle ABC)$

2. If $T_{3, -7}(TUVW) = T'U'V'W'$, what translation maps $T'U'V'W'$ onto $TUVW$?  
   - $\text{F} \circ T_{3, -7}(T'U'V'W')$
   - $\text{H} \circ T_{7, -3}(T'U'V'W')$
   - $\text{G} \circ T_{-7, 3}(T'U'V'W')$
   - $\text{I} \circ T_{-3, 7}(T'U'V'W')$

3. Which of the following is true for a rigid motion?  
   - $\text{A} \circ$ The preimage and the image have the same measurements.
   - $\text{B} \circ$ The preimage is larger than the image.
   - $\text{C} \circ$ The preimage is smaller than the image.
   - $\text{D} \circ$ The preimage is in the same position as the image.

4. $\triangle RSV$ has coordinates $R(2, 1), S(3, 2),$ and $V(2, 6)$. A translation maps point $R$ to $R'$ at $(-4, 8)$. What are the coordinates for $S'$ for this translation?  
   - $\text{F} \circ (-6, -4)$
   - $\text{G} \circ (-3, 2)$
   - $\text{H} \circ (-3, 9)$
   - $\text{I} \circ (-4, 13)$

Short Response

5. $\triangle LMP$ has coordinates $L(3, 4), M(6, 6),$ and $P(5, 5)$. A translation maps point $L$ to $L'$ at $(7, -4)$. What are the coordinates for $M'$ for this translation?  
   - $\text{[2]} M'(10, -2) \text{ and } P'(9, -3)$  
   - $\text{[1]}$ one correct coordinate given  
   - $\text{[0]}$ no correct coordinates given
9-1 Enrichment
Translations

Soccer Field Translations

Below is a diagram of half a soccer field with seven offensive players. By passing and dribbling the ball, the players try to score by getting the ball into the goal. When a player passes, dribbles, or kicks the ball, the ball is being translated on the soccer field.

Use the diagram to complete each Exercise. Each unit on the grid represents 1 unit on a coordinate grid. Assume the ball’s starting position is \((x, y)\).

1. Garcia passes the ball to Wilson. What is the rule for this translation? 
   \(T_{(-7, -2)}(x, y)\)
2. Young passes the ball to Foster. What is the rule for this translation? 
   \(T_{(4, -8)}(x, y)\)
3. Kwan passes the ball to Carmona. What is the rule for this translation? 
   \(T_{(-10, 7)}(x, y)\)
4. Adams passes the ball to Kwan, who then passes the ball to Garcia. Write two rules to describe these translations. 
   \(T_{(4, -8)}(x, y)\) and \(T_{(-7, -3)}(x, y)\)
5. Carmona passes the ball according to the rule \(T_{(-8, -6)}(x, y)\). Who receives Carmona’s pass? Foster
6. Wilson kicks the ball according to the rule \(T_{(3, -6)}(x, y)\). Is a goal scored if the goalie does not block the ball? Yes
7. Young dribbles the ball according to the rule \(T_{(0, -10)}(x, y)\), and then passes the ball according to the rule \(T_{(8, -4)}(x, y)\). Who receives Young’s pass? Wilson
8. If Carmona kicks the ball toward the goal and the goalie stops it in the penalty area, which of the following rules could represent Carmona’s kick? B
   - A: \(T_{(-6, -12)}(x, y)\)
   - B: \(T_{(-3, -15)}(x, y)\)
   - C: \(T_{(0, -12)}(x, y)\)
   - D: \(T_{(-5, -15)}(x, y)\)
A translation is a type of transformation. In a translation, a geometric figure changes position, but does not change shape or size. The original figure is called the *preimage* and the figure following transformation is the *image*.

The diagram at the right shows a translation in the coordinate plane. The preimage is \( \triangle ABC \). The image is \( \triangle A'B'C' \).

Each point of \( \triangle ABC \) has moved 5 units left and 2 units up. Moving left is in the negative \( x \) direction, and moving up is in the positive \( y \) direction. So each \((x, y)\) pair in \( \triangle ABC \) is mapped to \((x - 5, y + 2)\) in \( \triangle A'B'C' \). The function notation \( T_{-5, 2}\)(\( \triangle ABC \)) = \( \triangle A'B'C' \) describes this translation.

All translations are rigid motions because they preserve distance and angle measures.

**Problem**

What are the vertices of \( T_{-5, -1}\)(\( \triangle WXYZ \))?

Graph the image of \( \triangle WXYZ \).

\[
\begin{align*}
T_{-5, -1}(W) &= (-4 + 5, 1 - 1), \text{ or } W'(1, 0) \\
T_{-5, -1}(X) &= (-4 + 5, 4 - 1), \text{ or } X'(1, 3) \\
T_{-5, -1}(Y) &= (-1 + 5, 4 - 1), \text{ or } Y'(4, 3) \\
T_{-5, -1}(Z) &= (-1 + 5, 1 - 1), \text{ or } Z'(4, 0)
\end{align*}
\]

**Exercises**

Use the rule to find the vertices of the image.

1. \( T_{-2, -3}\)(\( \triangle MNO \))

\[
M'(0, 1), N'(-1, -2), O'(1, -3)
\]

2. \( T_{-1, 0}\)(\( \triangle JKL \))

\[
J'(-2, 2), K'(1, 2), L'(1, -1), M'(-2, -1)
\]
Problem

What rule describes the translation that maps $ABCD$ onto $A'B'C'D'$?

To get from $A$ to $A'$ (or from $B$ to $B'$ or $C$ to $C'$ or $D$ to $D'$), you move 8 units left and 7 units down. The translation maps $(x, y)$ to $(x - 8, y - 7)$. The translation rule is $T_{-8, -7}(ABCD)$.

Exercises

- On graph paper, draw the $x$- and $y$-axes, and label Quadrants I–IV.
- Draw a quadrilateral in Quadrant I. Make sure that the vertices are on the intersection of grid lines.
- Trace the quadrilateral, and cut out the copy.
- Use the cutout to translate the figure to each of the other three quadrants.

Write the rule that describes each translation of your quadrilateral.

3. from Quadrant I to Quadrant II  
   Check students’ work.
4. from Quadrant I to Quadrant III  
   Check students’ work.
5. from Quadrant I to Quadrant IV  
   Check students’ work.
6. from Quadrant II to Quadrant III  
   Check students’ work.
7. from Quadrant III to Quadrant I  
   Check students’ work.

Refer to $ABCD$ in the problem above.

8. Give the vertices of $T_{-2, -3}(ABCD)$. $A'(0, -3), B'(1, 1), C'(4, -1), D'(5, -4)$
9. Give the vertices of $T_{-2, -4}(ABCD)$. $A'(4, -2), B'(5, 2), C'(8, 0), D'(9, -3)$
10. Give the vertices of $T_{1, 3}(ABCD)$. $A'(3, 5), B'(4, 9), C'(7, 7), D'(8, 4)$
Additional Vocabulary Support

9-2

Reflections

Use the chart below to review vocabulary. These vocabulary words will help you complete this page.

<table>
<thead>
<tr>
<th>Related Words</th>
<th>Explanations</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orient</td>
<td>To place or direct in a position, with reference to a certain point</td>
<td>Orient the box so that the top faces up.</td>
</tr>
<tr>
<td>Orientation</td>
<td>The way an object is positioned</td>
<td>The orientation of the window is facing east.</td>
</tr>
<tr>
<td>Visual</td>
<td>A picture or display used to show something</td>
<td>A road map is a visual that can be used to figure out how to get to different places.</td>
</tr>
<tr>
<td>Vision</td>
<td>Seeing with the eyes, sight</td>
<td>His vision is so good he can read the street sign from 60 feet away.</td>
</tr>
<tr>
<td>Visualize</td>
<td>To form a picture of something in your mind</td>
<td>An artist tries to visualize a finished painting before starting to paint.</td>
</tr>
<tr>
<td>Reflect</td>
<td>To show an image of something</td>
<td>The mirror reflects light onto the wall.</td>
</tr>
<tr>
<td>Reflection</td>
<td>A mirror image of an object</td>
<td>The artist can see a reflection of the sky on the pond.</td>
</tr>
</tbody>
</table>

Circle the correct word to complete the sentence.

1. The [orient/orientation] of the solar panel is toward the sun.
2. The water [reflects/reflection] the clouds and trees around the lake.
3. The editor can read small print. He has good [visual/vision].

Use the vocabulary above to fill in the blanks.

4. He learns a lot from pictures. He is a visual learner.
5. She looks at her reflection in the mirror as she combs her hair.
6. Before she plays in the game, the soccer player will visualize herself scoring a goal.
9-2  Think About a Plan

Reflections

Copy the pair of figures. Then draw the line of reflection you can use to map one figure onto the other. *Answers may vary. Samples:*

1. Designate one figure to be the preimage and one to be its reflection. Label two points $A$ and $B$ on the preimage and label their images ($A'$ and $B'$) on the reflection.

2. Sketch $AA'$ and $BB'$. This step is optional, but provides a frame of reference for later steps.

3. Open your compass to a size greater than half the distance between point $A$ and $A'$. With the compass at $A$, draw an arc above and below the approximate midpoint of $AA'$.

4. Do the same from point $A'$. Label points $P$ and $Q$ at the points where the arcs intersect.

5. Draw $PQ$. To check your work, do the same from points $B$ and $B'$. 

*Answers may vary. Samples:
Find the coordinates of each image.

1. \(R_x\)-axis(A) \((2, -2)\)
2. \(R_y\)-axis(B) \((-4, 4)\)
3. \(R_y = 1\)(C) \((2, 4)\)
4. \(R_x = -1\)(D) \((0, 2)\)
5. \(R_y = -1\)(E) \((-3, 0)\)
6. \(R_x = 2\)(F) \((5, -3)\)

Coordinate Geometry  Given points \(M(3, 3)\), \(N(5, 2)\), and \(O(4, 4)\), graph \(\triangle MNO\) and its reflection image as indicated.

7. \(R_y\)-axis

8. \(R_x\)-axis

9. \(R_x = 1\)

10. \(R_y = -2\)

Copy each figure and line \(\ell\). Draw each figure’s reflection image across line \(\ell\).

11. 

12. 

13. 

14. 

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Copy each pair of figures. Then draw the line of reflection you can use to map one figure onto the other.

15.  

16.  

Find the image of $Z(1, 1)$ after two reflections, first across line $\ell_1$, and then across line $\ell_2$.

17. $\ell_1 : x = 2, \ell_2 : y$-axis $(-3, 1)$  
18. $\ell_1 : x = -2, \ell_2 : x$-axis $(-5, -1)$  
19. $\ell_1 : y = 2, \ell_2 : x$-axis $(1, -3)$  
20. $\ell_1 : y = -3, \ell_2 : y$-axis $(-1, -7)$  
21. $\ell_1 : x = 3, \ell_2 : y = 2 (5, 3)$  
22. $\ell_1 : x = -1, \ell_2 : y = -3 (-3, -7)$

Use the graph at the right for Exercises 23 and 24.

23. Berit lives 3 mi east of Rt. 147 and 1 mi north of Rt. 9. Jane lives 3 mi east of Rt. 147 and 5 mi north of Rt. 9. The girls want to start at Berit’s house, hike to Rt. 147, then on to Jane’s house. They want to hike the shortest distance possible. To which point on Rt. 147 should they walk? (Hint: First find the line of reflection if Berit’s house is reflected onto Jane’s house.)
   to a point 3 mi north of Rt. 9

24. Instead of ending the hike at Jane’s house, the girls want to hike to an inn 2 mi north of Jane’s house. They want to hike the shortest possible total distance, starting from Berit’s house, walking to Rt. 147, and then to the inn. To which point on Rt. 147 should they walk? (Hint: First find the line of reflection if Berit’s house is reflected onto the inn.)
   to a point 4 mi north of Rt. 9

25. Point $A$ on a coordinate grid is at $(3, 4)$. What are the coordinates of $R_{y=\frac{1}{2}}(A)?? (4, 3)$

26. Point $Z$ on a coordinate grid is at $(-1, 3)$. What are the coordinates of $R_{y=-\frac{1}{2}}(Z)$? $(-3, 1)$

27. Give an example of a place you may see a geometric reflection in everyday life. Explain.  
   Answers may vary. Samples: patterns in tiles, wallpaper, paintings
9-2 Practice

Reflections

Find the coordinates of each image.

1. \( R_{x}\)-axis(\(A\)) \((4, -2)\)
2. \( R_{y}\)-axis(\(B\)) \((-3, -3)\)
3. \( R_{y} = -1\)(\(C\)) \((-2, -1)\)
4. \( R_{x} = -1\)(\(D\)) \((2, -3)\)
5. \( R_{y} = -3\)(\(E\)) \((1, -5)\)
6. \( R_{x} = -2\)(\(F\)) \((-6, 3)\)

Coordinate Geometry
Given points \(S(1, -2)\), \(T(4, -1)\), and \(V(4, -4)\), graph \(\triangle STV\) and its reflection image as indicated.

7. \( R_{y}\)-axis
   To start, draw the triangle and show the \(y\)-axis as the dashed line of reflection.
   Then locate \(S'\) so that the \(y\)-axis is the perpendicular bisector of \(SS'\).
   Repeat to find \(T'\) and \(V'\).

8. \( R_{x}\)-axis

   Copy each figure and line \(\ell\). Draw each figure’s reflection image across line \(\ell\).

9. \( R_{x} = -1\)

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Copy each pair of figures. Then draw the line of reflection you can use to map one figure onto the other.

12.  

13.  

Use the figure at the right to help you solve Exercises 14 and 15.

14. Shari wants to start at school, walk to Main St., then continue on to her house. She wants the whole trip to be the shortest distance possible. To which point on Main St. should she walk?

To start, draw a line of reflection $y = 4$.

Where does the line of reflection intersect Main St.? at $(0, 4)$

On the graph, draw the shortest route from school to Main St. to Shari's house.

15. Shari decides instead to walk to the library. She wants to walk the shortest possible total distance, starting from school, walking to Main St., and then to the library. On the graph, draw a line of reflection to help you find the shortest path. Then draw the path she should take to the library.

For Exercises 16–20, find the coordinates of each image.

16. $R_{y=x}(U)$ $U'(1, 4)$

To start, draw line $\ell_1$ through $U$ perpendicular to $y = x$.

The slope of $y = x$ is $1$, so the slope of line $\ell_1$ is $-1$.

What are the coordinates of $U'$ so that $UU'$ is bisected by $y = x$? $1; -1; U'(1, 4)$

17. $R_{y=x}(V)$ $V'(1, 2)$

18. $R_{y=x}(W)$ $W'(-2, 2)$

19. $R_{y=x}(X)$ $X'(-3, -1)$

20. $R_{y=x}(Y)$ $Y'(-5, -2)$
Multiple Choice

For Exercises 1–5, choose the correct letter.

1. In the graph at the right, what are the coordinates of $R_y(D)$?
   - (A) $(3, 1)$
   - (B) $(3, -1)$
   - (C) $(3, -1)$
   - (D) $(3, 1)$

2. The coordinates of the vertices of $\triangle CDE$ are $C(1, 4)$, $D(3, 6)$, and $E(7, 4)$. What are the coordinates of $R_{y=3}(D)$?
   - (F) $(3, -6)$
   - (G) $(3, -3)$
   - (H) $(3, 0)$
   - (I) $(3, 9)$

3. What is true for an image and a preimage in a reflection?
   - (A) The image is larger than the preimage.
   - (B) The image is smaller than the preimage.
   - (C) The image and the preimage have the same orientation.
   - (D) The image and the preimage have different orientations.

4. In the graph at the right, what is the line of reflection for $\triangle XYZ$ and $\triangle X’Y’Z’$?
   - (F) the $x$-axis
   - (G) the $y$-axis
   - (H) $x = 2$
   - (I) $y = 2$

5. What is the image of $A(3, -1)$ after a reflection, first across the line $y = 3$, and then across the line $x = -1$?
   - (A) $(-5, 7)$
   - (B) $(3, -1)$
   - (C) $(-5, -1)$
   - (D) $(1, -5)$

Extended Response

6. The coordinates of the vertices of parallelogram $RSTU$ are $R(0, 0)$, $S(2, 3)$, $T(6, 3)$, and $U(4, 0)$. What are the coordinates of the vertices of $R_y(RSTU)$?
   - $[4]$ $R’(0, 0)$, $S’(3, 2)$, $T’(3, 6)$, and $U’(0, 4)$
   - [3] Any three of the four coordinates are correct.
   - [2] Any two of the four coordinates are correct.
   - [1] Any one of the four coordinates is correct.
   - [0] No correct coordinates are given.
9-2 Enrichment

Reflections

Lines of Reflection

A line of reflection is the line across which the preimage is reflected to produce the image. When a reflection is shown on a grid, you can use algebra to locate and give the equation of the line of reflection.

Use the figure at the right to complete Exercises 1–6.

1. What point is halfway between $J$ and $J'$? $(-3, -1)$
2. What point is halfway between $K$ and $K'$? $(-1, 0)$
3. What point is halfway between $L$ and $L'$? $(5, 3)$
4. What formula did you use in Exercises 1–3?
   
   **the Midpoint Formula**
5. Use the points found in Exercises 1–3 to draw the line of reflection.
6. What is the equation of the line of reflection? Use slope-intercept form. $y = \frac{1}{2}x + \frac{1}{2}$

Draw the line of reflection on each grid. Then give the equation of the line of reflection.

7. $y = -4$
8. $y = -x$
9. $x = 1$
10. $y = -2x - 2$
11. $y = 3x + 2$
12. $y = -\frac{1}{2}x - 1$
A \textit{reflection} is a type of transformation in which a geometric figure is flipped across a \textit{line of reflection}.

In a reflection, a \textit{preimage} and an \textit{image} have opposite orientations, but are the same shape and size. Because a reflection preserves both distance and angle measure, a reflection is a \textit{rigid motion}.

Using function notation, the reflection across line \( m \) can be written as \( R_m \).

For example, if \( P' \) is the image of \( P \) reflected over the line \( x = 1 \), then \( R_{x=1}(P) = P' \).

\textbf{Problem}

What are the reflection images of \( \triangle MNO \) across \( x \)- and \( y \)-axes? Give the coordinates of the vertices of \( R_{x\text{-axis}}(\triangle MNO) \) and \( R_{y\text{-axis}}(\triangle MNO) \).

The coordinates of the vertices of \( R_{x\text{-axis}}(\triangle MNO) \) are \((2, -3)\), \((3, -7)\), and \((5, -4)\). The coordinates of the vertices of \( R_{y\text{-axis}}(\triangle MNO) \) are \((-2, 3)\), \((-3, 7)\), and \((-5, 4)\).

\textbf{Exercises}

Use a sheet of graph paper to complete Exercises 1–5.

1. Draw and label the \( x \)- and \( y \)-axes on a sheet of graph paper. \textbf{Check students’ work.}

2. Draw a scalene triangle in one of the four quadrants. Make sure that the vertices are on the intersection of grid lines. \textbf{Check students’ work.}

3. Fold the paper along the axes. \textbf{Check students’ work.}

4. Cut out the triangle, and unfold the paper. \textbf{Check students’ work.}

5. Label the coordinates of the vertices of the reflection images across the \( x \)- and \( y \)-axes. \textbf{Check students’ work.}
9-2 Reteaching (continued)

Refl ections

To graph a refl ection image on a coordinate plane, graph the images of each vertex. Each vertex in the image must be the same distance from the line of refl ection as the corresponding vertex in the preimage.

<table>
<thead>
<tr>
<th>$R_{x\text{-axis}}$ describes the reflection across the $x$-axis.</th>
<th>$R_{y\text{-axis}}$ describes the reflection across the $y$-axis.</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $P$ has coordinates $(x, y)$ and $R_{x\text{-axis}}(P) = P'$, then $P'$ has coordinates $(x, -y)$.</td>
<td>If $P$ has coordinates $(x, y)$ and $R_{y\text{-axis}}(P) = P'$, then $P'$ has coordinates $(-x, y)$.</td>
</tr>
<tr>
<td>The $x$-coordinate does not change.</td>
<td>The $y$-coordinate does not change.</td>
</tr>
<tr>
<td>The $y$-coordinate tells the distance from the $x$-axis.</td>
<td>The $x$-coordinate tells the distance from the $y$-axis.</td>
</tr>
</tbody>
</table>

Problem

$\triangle ABC$ has vertices at $A(2, 4)$, $B(6, 4)$, and $C(3, 1)$. What is the graph of $R_{x\text{-axis}}(\triangle ABC)$?

Step 1: Graph $A'$, the image of $A$. It is the same distance from the $x$-axis as $A$. The distance from the $y$-axis has not changed. The coordinates for $A'$ are $(2, -4)$.

Step 2: Graph $B'$. It is the same distance from the $x$-axis as $B$. The distance from the $y$-axis has not changed. The coordinates for $B'$ are $(6, -4)$.

Step 3: Graph $C'$. It is the same distance from the $x$-axis as $C$. The coordinates for $C'$ are $(3, -1)$.

Step 4: Draw $\triangle A'B'C'$.

Each figure is reflected as indicated. Find the coordinates of the vertices for each image.

6. $\triangle FGH$ with vertices $F(-1, 3)$, $G(-5, 1)$, and $H(-3, 5)$ reflected by $R_{x\text{-axis}}$
   $F'(-1, -3)$, $G'(-5, -1)$, $H'(-3, -5)$
7. $\triangle CDE$ with vertices $C(2, 4)$, $D(5, 2)$, and $E(6, 3)$ reflected by $R_{x\text{-axis}}$
   $C'(2, -4)$, $D'(5, -2)$, $E'(6, -3)$
8. $\triangle JKL$ with vertices $J(-1, -5)$, $K(-2, -3)$, and $L(-4, -6)$ reflected by $R_{y\text{-axis}}$
   $J'(1, -5)$, $K'(2, -3)$, $L'(4, -6)$
9. Quadrilateral $WXYZ$ with vertices $W(-3, 4)$, $X(-4, 6)$, $Y(-2, 6)$, and $Z(-1, 4)$ reflected by $R_{y\text{-axis}}$
   $W'(3, 4)$, $X'(4, 6)$, $Y'(2, 6)$, $Z'(1, 4)$
### Concept List

<table>
<thead>
<tr>
<th>angle of rotation</th>
<th>center of rotation</th>
<th>clockwise rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>clockwise</td>
<td>rigid motion</td>
<td>rotation</td>
</tr>
<tr>
<td>counterclockwise</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Choose the concept from the list above that best represents the item in each box.

1. Point X
   - [Diagram of a point labeled X with a diamond around it.]
   - center of rotation

2. These transformations are all examples of this.
   - [Diagram of a star being rotated.]
   - rigid motion

3. This type of transformation
   - [Diagram of a diamond being rotated.]
   - rotation

4. $90^\circ$ for this transformation
   - [Diagram of a square being rotated by $90^\circ$.]
   - angle of rotation

5. This transformation maps each point $(x, y)$ to $(y, -x)$. $r_{(270^\circ, O)}$
   - [Diagram of a square being rotated by $270^\circ$.]

6. [Diagram of a circle rotating clockwise.]
   - clockwise

7. [Diagram of a circle rotating counterclockwise.]
   - counterclockwise

8. This transformation maps each point $(x, y)$ to $(-y, x)$. $r_{(90^\circ, O)}$
   - [Diagram of a square being rotated by $90^\circ$.]

9. This transformation maps each point $(x, y)$ to $(-x, -y)$. $r_{(180^\circ, O)}$
   - [Diagram of a square being rotated by $180^\circ$.]
Think About a Plan

Rotations

Coordinate Geometry  Graph $A(5, 2)$. Graph $B$, the image of $A$ for a $90^\circ$ rotation about the origin $O$. Graph $C$, the image of $A$ for a $180^\circ$ rotation about $O$. Graph $D$, the image of $A$ for a $270^\circ$ rotation about $O$. What type of quadrilateral is $ABCD$? Explain.

1. To find the coordinates of $B$, you can use the rule for rotating a point $90^\circ$ about the origin $O$ in a coordinate plane. What are the coordinates of $B = r_{(90^\circ, O)}(A)$?
   $B(-2, 5)$

2. To find the coordinates of $C$, you can use the rule for rotating a point $180^\circ$ about the origin $O$ in a coordinate plane. What are the coordinates of $C = r_{(180^\circ, O)}(A)$?
   $C(-5, -2)$

3. To find the coordinates of $D$, you can use the rule for rotating a point $270^\circ$ about the origin $O$ in a coordinate plane. What are the coordinates of $D = r_{(270^\circ, O)}(A)$?
   $D(2, -5)$

4. Graph $A$, $B$, $C$, and $D$.


6. The figure appears to be a square. How can you show that the figure is a square?
   Answers may vary. Sample: Show that $AB = BC = CD = DA$ and $m\angle A = m\angle B = m\angle C = m\angle D = 90^\circ$.

7. Using the Distance Formula, what are $AB$, $BC$, $CD$, and $DA$?
   $AB = BC = CD = DA = \sqrt{58}$

8. What are the slopes of $AB$, $BC$, $CD$, and $DA$?
   $slop AB = slope CD = -\frac{3}{7}; slope BC = slope DA = \frac{7}{3}$

9. What can you conclude about $\angle A$, $\angle B$, $\angle C$, and $\angle D$? Explain.
   They are right angles, because $AB \perp BC$, $BC \perp CD$, $CD \perp DA$, and $DA \perp AB$. 

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Copy each figure and point $P$. Draw the image of each figure for the given rotation about $P$. Use prime notation to label the vertices of the image.

1. $60^\circ$
   ![Diagram 1]
   $A'B'C'D'$

2. $90^\circ$
   ![Diagram 2]
   $A'B'C'$

3. $120^\circ$
   ![Diagram 3]
   $A'B'C'$

4. $180^\circ$
   ![Diagram 4]
   $A'B'C'$

Copy each figure and point $P$. Then draw the image of $JK$ for a $180^\circ$ rotation about $P$. Use prime notation to label the vertices of the image.

5. $J$
   ![Diagram 5]
   $J'K'$

6. $J$
   ![Diagram 6]
   $J'K'$

Point $O$ is the center of regular hexagon $BCDEFG$. Find the image of the given point or segment for the given rotation.

7. $r_{(120^\circ, O)}(F)$
   $D$

8. $r_{(180^\circ, O)}(B)$
   $E$

9. $r_{(300^\circ, O)}(BG)$
   $CB$

10. $r_{(360^\circ, O)}(CD)$
    $CD$

11. $r_{(60^\circ, O)}(E)$
    $D$

12. $r_{(240^\circ, O)}(FE)$
    $BG$
For Exercises 13–15, \( \triangle ABC \) has vertices \( A(2, 2), B(3, -2), \) and \( C(-1, 3) \).

13. Graph \( r_{(90^\circ, O)}(\triangle ABC) \).

14. Graph \( r_{(180^\circ, O)}(\triangle ABC) \).

15. Graph \( r_{(270^\circ, O)}(\triangle ABC) \).

16. The vertices of \( PQRS \) have coordinates \( P(-1, 5), Q(3, 4), R(2, -4), \) and \( S(-3, -2) \). What are the coordinates of the vertices of \( r_{(270^\circ, O)}(PQRS) \)?

17. The vertices of \( r_{(90^\circ, O)}(KLNM) \) have coordinates \( K'(-3, 2), L'(2, 3), M'(4, -2), \) and \( N'(-2, -4) \). What are the coordinates of the vertices of \( KLNM \)?

18. **Reasoning** The vertices of quadrilateral \( ABCD \) have coordinates \( A(4, 3), B(-3, 4), C(-4, -3), \) and \( D(3, -4) \). Explain how the transformation \( r_{(90^\circ, O)}(ABCD) = BCDA \) can be used to show that the quadrilateral is square.

Answers may vary. Sample: Because the transformation is a rigid motion, \( AB = BC = CD = DA \) and \( m\angle A = m\angle B = m\angle C = m\angle D \). Since all angles of the quadrilateral have the same measure, the angle measure of each angle must be \( 90^\circ \). Thus, the quadrilateral is a square by definition.

Find the angle of rotation about \( D \) that maps the solid-line figure to the dashed-line figure.

19. 

20. about \( 340^\circ \)

21. A pie is cut into 12 equal slices. What is the angle of rotation about the center that will map a piece of pie to a piece that is two slices away from it? \( 60^\circ \)

22. \( \triangle RST \) has vertices at \( R(0, 3), S(4, 0), \) and \( T(0, 0) \). What are the coordinates of the vertices of \( r_{(90^\circ, T)}(\triangle RST) \)?

23. \( \triangle FGH \) has vertices \( F(-1, 2), G(0, 0), \) and \( H(3, -1) \). What are the coordinates of the vertices of \( r_{(90^\circ, G)}(\triangle FGH) \)?
Copy each figure and point $R$. Draw the image of each figure for the given rotation about $R$. Use prime notation to label the vertices of the image.

1. $30^\circ$
   To start, draw a $30^\circ$ angle with $R$ as the vertex and $\overline{RA}$ as one side.
   Locate $A'$ so that $\overline{RA'} \cong 90^\circ \cdot \overline{RA}$.
   Continue to find $B'$ and $C'$.

2. $60^\circ$

3. $90^\circ$

Copy each figure and point $R$. Then draw the image of $\overline{XY}$ for a $120^\circ$ rotation about $R$. Use prime notation to label the vertices of the image.

4.

5.

Point $R$ is the center of regular quadrilateral $MATH$. Find the image of the given point or segment for the given rotation.

6. $r_{(90^\circ, R)}(H) \rightarrow T$

7. $r_{(180^\circ, R)}(M) \rightarrow T$

8. $r_{(270^\circ, R)}(AT) \rightarrow TH$

9. $r_{(360^\circ, R)}(HM) \rightarrow HM$
For Exercises 10–12, $ABCD$ has vertices $A(1, 1)$, $B(1, 3)$, $C(4, 3)$, and $D(4, 1)$.

10. Graph $r_{(90^\circ, 0)}(ABCD)$.
   To start, graph $ABCD$.
   $A' = r_{(90^\circ, 0)}(A) = (-1, 1)$
   $B' = r_{(90^\circ, 0)}(B) = (-3, 1)$
   $C' = r_{(90^\circ, 0)}(C) = (-3, 4)$
   $D' = r_{(90^\circ, 0)}(D) = (-1, 4)$
   Then graph $A'$, $B'$, $C'$, and $D'$.

11. Graph $r_{(180^\circ, 0)}(ABCD)$.

12. Graph $r_{(270^\circ, 0)}(ABCD)$.

13. The vertices of $\triangle PQR$ have coordinates $P(1, 5)$, $Q(3, 1)$, and $R(-2, 1)$. What are the coordinates of the vertices of $r_{(90^\circ, 0)}(\triangle PQR)$?
   $P'(-5, 1)$, $Q'(-1, 3)$, $R'(-1, -2)$

14. $ABCD$ has vertices $A(4, 2)$, $B(-2, 2)$, $C(-4, -2)$, and $D(2, -2)$. Which of the following quadrilaterals is $r_{(180^\circ, 0)}(ABCD)$?
   - $A\triangleright ABCD$
   - $B\triangleright BCDA$
   - $C\triangleright CDAB$
   - $D\triangleright DABC$

Find the angle of rotation about $B$ that maps the solid-line figure to the dashed-line figure.

15. $270^\circ$

16. $120^\circ$

17. $\triangle XYZ$ has vertices at $X(2, 0)$, $Y(0, 0)$, and $Z(0, 5)$. Find the coordinates of the vertices of $r_{(180^\circ, \gamma)}(\triangle XYZ)$.
   $X'(-2, 0)$, $Y'(0, 0)$, $Z'(0, -5)$
Multiple Choice

In Exercises 1–5, choose the correct letter.

1. Square $ABCD$ has vertices $A(3, 3), B(-3, 3), C(-3, -3),$ and $D(3, -3)$.
Which of the following images is $A$?
- $\text{A} \rightarrow r_{(90^\circ, O)}(C)$
- $\text{B} \rightarrow r_{(180^\circ, O)}(D)$
- $\text{C} \rightarrow r_{(270^\circ, O)}(B)$
- $\text{D} \rightarrow r_{(270^\circ, O)}(C)$

2. $r_{(90^\circ, O)}(PQRS)$ has vertices $P'(1, 5), Q'(3, -2), R'(-3, -2),$ and $S'(-5, 1)$.
What are the coordinates of $Q$?
- $\text{A} \rightarrow (-2, -3)$
- $\text{B} \rightarrow (-3, 2)$
- $\text{C} \rightarrow (2, 3)$
- $\text{D} \rightarrow (-3, -2)$

3. Point $A$ is the center of regular hexagon $GHIJKL$.
What is $r_{(300^\circ, A)}(I)$?
- $\text{A} \rightarrow J$
- $\text{B} \rightarrow K$
- $\text{C} \rightarrow L$
- $\text{D} \rightarrow M$

4. A Ferris wheel has 16 cars spaced equal distances apart. The cars are numbered 1–16 clockwise. What is the measure of the angle of rotation that will map the position of car 16 onto the position of car 13?
- $\text{F} \rightarrow 22.5^\circ$
- $\text{G} \rightarrow 45^\circ$
- $\text{H} \rightarrow 67.5^\circ$
- $\text{I} \rightarrow 90^\circ$

5. Given $P(2, -5)$, what are the coordinates of $r_{(90^\circ, O)}(P)$?
- $\text{A} \rightarrow (5, 2)$
- $\text{B} \rightarrow (-5, 2)$
- $\text{C} \rightarrow (5, -2)$
- $\text{D} \rightarrow (-2, -5)$

Short Response

6. $\triangle ABC$ has coordinates $A(3, 3), B(0, 0),$ and $C(3, 0)$. What are the coordinates of $r_{(180^\circ, B)}(A)$ and $r_{(180^\circ, B)}(C)$?

- $[2] A'(-3, -3)$ and $C'(-3, 0)$ [1] one correct coordinate given [0] no correct coordinates given
Three-Dimensional Rotations

Rotations in three dimensions are generally covered in calculus and analytical geometry. However, a little deeper insight into rotations in two dimensions can be gained by taking a brief glimpse at rotations in three dimensions.

The three-dimensional coordinate system has three axes: \(x\), \(y\), and \(z\), as shown at the right. There are three intersecting planes, each of which is referred to by the two axes that it contains: the \(xy\) plane, the \(xz\) plane, and the \(yz\) plane. Each point in the system has coordinates in the form \((x, y, z)\).

In two dimensions, rotations are movements around a fixed point in a plane. A triangle rotated about the origin in two dimensions could have been “moved” just as easily by leaving the triangle as is and rotating the \(x\)- and \(y\)-axes. In fact, in three dimensions this is exactly how rotations are accomplished. The axes are rotated, and the geometric figure is mapped to a new location.

For example, the rectangular solid below has vertices at \(A(-2, 0, 3)\), \(B(-2, 2, 3)\), \(C(-2, 2, 0)\), \(D(-2, 0, 0)\), \(E(-6, 0, 0)\), \(F(-6, 0, 3)\), \(G(-6, 2, 3)\), and \(H(-6, 2, 0)\).

The second figure shows the solid mapped to a new position by a clockwise 90° rotation of the \(x\)- and \(y\)-axes.

What are the coordinates of the new vertices?

1. \(A' (0, -2, 3)\)
2. \(B' (-2, -2, 3)\)
3. \(C' (-2, -2, 0)\)
4. \(D' (0, -2, 0)\)
5. \(E' (0, -6, 0)\)
6. \(F' (0, -6, 3)\)
7. \(G' (-2, -6, 3)\)
8. \(H' (-2, -6, 0)\)

What will the coordinates be if the original rectangular solid is mapped to a new position by a clockwise 180° rotation of the \(x\)- and \(y\)-axes?

9. \(A' (2, 0, 3)\)
10. \(B' (2, -2, 3)\)
11. \(C' (2, -2, 0)\)
12. \(D' (2, 0, 0)\)
13. \(E' (6, 0, 0)\)
14. \(F' (6, 0, 3)\)
15. \(G' (6, -2, 3)\)
16. \(H' (6, -2, 0)\)
9-3 Reteaching
Rotations

A turning of a geometric figure about a point is a rotation. The center of rotation is the point about which the figure is turned. The number of degrees the figure turns is the angle of rotation. (In this chapter, rotations are counterclockwise unless otherwise noted.)

A rotation is a rigid motion because it preserves distance and angle measure. Using function notation, the rotation of $x^8$ with center of rotation $P$ can be written as $r(x^8, P)$.

For example, $ABCD$ is rotated about $Z$. $ABCD$ is the preimage and $A'B'C'D'$ is the image. The center of rotation is point $Z$. The angle of rotation is $82^\circ$. Using function notation, $r(82^\circ, Z)(ABCD) = A'B'C'D'$.

The distance from the center of rotation to a point in the preimage is the same as the distance from the center of rotation to the corresponding image point. The measure of the angle formed by a point in the image, the center of rotation as the vertex, and the corresponding image point is equal to the angle of rotation. In the example, $ZA =ZA', ZB =ZB', ZC =ZC', ZD =ZD'$, and $m\angle AZA' = m\angle BZB' = m\angle CZC' = m\angle DZD' = 82^\circ$.

Exercises

Complete the following steps to draw $r_{(80^\circ, T)}(\triangle XYZ)$.

1. Draw $TX$. Use a protractor to draw an $80^\circ$ angle with vertex $T$ and side $TX$.

2. Use a compass to construct $TX' \equiv TX$.

3. Locate $Y'$ and $Z'$ in a similar manner.

4. Draw $\triangle X'Y'Z'$.

Copy $\triangle XYZ$ to complete Exercises 5–7.

5. Draw $r_{(120^\circ, T)}(\triangle XYZ)$.

6. Draw a point $S$ inside $\triangle XYZ$. Draw $r_{(135^\circ, S)}(\triangle XYZ)$.

7. Draw $r_{(90^\circ, Y)}(\triangle XYZ)$. 

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For a point \( P(x, y) \) in the coordinate plane, use the following rules to find the coordinates of the 90°, 180°, and 270° rotations about the origin \( O \).

\[
\begin{align*}
r_{(90^\circ, O)}(P) &= (-y, x) \\
r_{(180^\circ, O)}(P) &= (-x, -y) \\
r_{(270^\circ, O)}(P) &= (y, -x)
\end{align*}
\]

**Problem**

\( \triangle FGH \) has vertices \( F(1, 1), \ G(2, 3), \) and \( H(4, -2) \). Graph \( \triangle FGH \) and \( r_{(270^\circ, O)}(\triangle FGH) \).

First, plot \( F, \ G, \) and \( H \) and connect the points to graph \( \triangle FGH \).

Next, use the rule for a 270° rotation to find the coordinates of the vertices of \( r_{(270^\circ, O)}(\triangle FGH) = \triangle F'G'H' \).

\[
\begin{align*}
r_{(270^\circ, O)}(F) &= (1, -1) = F' \\
r_{(270^\circ, O)}(G) &= (3, -2) = G' \\
r_{(270^\circ, O)}(H) &= (-2, -4) = H'
\end{align*}
\]

Then plot \( F', \ G', \) and \( H' \) and connect the points to graph \( r_{(270^\circ, O)}(\triangle FGH) \).

**Exercises**

For Exercises 8 and 9, \( \triangle ABC \) has vertices \( A(2, 1), \ B(2, 3), \) and \( C(4, 1) \).

8. Graph \( r_{(90^\circ, O)}(\triangle ABC) \).

9. Graph \( r_{(180^\circ, O)}(\triangle ABC) \).

10. The vertices of \( \triangle DEF \) have coordinates \( D(-1, 2), E(3, 3), \) and \( F(2, -4) \). What are the coordinates of the vertices of \( r_{(90^\circ, O)}(\triangle DEF) \)?

\[
\begin{align*}
D' &= (-1, -2) \\
E' &= (-3, 3) \\
F' &= (4, 2)
\end{align*}
\]

11. The vertices of \( PQRS \) have coordinates \( P(-2, 3), Q(4, 3), R(4, -3), \) and \( S(-2, -3) \). What are the coordinates of the vertices of \( r_{(270^\circ, O)}(PQRS) \)?

\[
\begin{align*}
P' &= (3, -2) \\
Q' &= (3, -4) \\
R' &= (-3, -4) \\
S' &= (-3, 2)
\end{align*}
\]
A student wants to find the image of \( \triangle ABC \) for the glide reflection \( R_{x=1} \circ T_{<0,2>}. \)

He wrote these steps to solve the problem on note cards, but they got mixed up.

Use the note cards to write the steps in order. The completed graph is shown below right.

1. First, write the rule to use for the translation. \( T_{<0,2>} \) maps each \((x, y)\) to \((x, y + 2)\).

2. Second, find the translation image. The image of \( A \) is at \((-2, 3)\), the image of \( B \) is at \((-4, 1)\), and the image of \( C \) is at \((-1, 1)\).

3. Next, draw the translation image of \( \triangle ABC \).

4. Then, graph the line of reflection \( x = 1 \).

5. Then, find the images of \( A, B, \) and \( C \) for a reflection of the translation image.

6. Finally, draw the reflection image of \( \triangle ABC \).
Identify the mapping $\triangle EDC \rightarrow \triangle PQM$ as a translation, reflection, rotation, or glide reflection. Write the rule for each translation, reflection, rotation, or glide reflection. For glide reflections, write the rule as a composition of a translation and reflection.

**Know**

1. What strategy could you use to isolate the figures with which you are working?

   Draw a diagram of $\triangle EDC$ and $\triangle PQM$ only.

2. What kind of mapping appears to have happened here? Explain.

   a glide reflection; $\triangle EDC$ is translated from Quadrant II to Quadrant I, and then reflected into Quadrant IV on to $\triangle PQM$

**Need**

3. What more do we need to know to have a complete answer?

   a translation rule and a reflection rule

**Plan**

4. Write a translation rule that maps $\triangle EDC$ onto $\triangle PQM$.

   Answers may vary. Sample: $T_{<11,0>}$

5. Write a reflection rule that maps $\triangle EDC$ onto $\triangle PQM$.

   Answers may vary. Sample: $R_{x-axis}$

6. Write a rule that maps $\triangle EDC$ onto $\triangle PQM$.

   Answers may vary. Sample: $(R_{x-axis} \circ T_{<11,0>})(\triangle EDC)$
Find the image of each letter after the transformation $R_m \circ R_\ell$. Is the resulting transformation a translation or a rotation? For a translation, describe the distance and direction. For a rotation, tell the center of rotation and the angle of rotation.

1. \[ \text{X} \quad \ell \quad m \]
   **translation; directly down; twice the distance between the two lines.**

Graph $\triangle DML$ and its glide reflection image.

3. $(R_y\text{-axis} \circ T_{<3,0>})(\triangle DML)$

5. $(R_x<2 \circ T_{<0,1>})(\triangle DML)$

7. Lines $\ell$ and $m$ intersect at point $P$ and are perpendicular. If a point $Q$ is reflected across $\ell$ and then across $m$, what transformation rule describes this composition? $r_{(180^\circ, P)}(Q)$

8. A triangle is reflected across line $\ell$ and then across line $m$. If this composition of reflections is a translation, what is true about $m$ and $\ell$? $\ell \parallel m$
Graph $\overline{AB}$ and its image $\overline{A'B'}$ after a reflection first across $\ell_1$ and then across $\ell_2$. Is the resulting transformation a translation or a rotation? For a translation, describe the direction and distance. For a rotation, tell the center of rotation and the angle of rotation.

9. $A(-3, 4), B(-1, 0)$; $\ell_1 : x = 1$; a $180^\circ$ rotation about $(1, -1)$ $\ell_2 : y = -1$

10. $A(-5, 2), B(-3, 6)$; $\ell_1 : x = -2$; a translation; 10 units to the right $\ell_2 : x = 3$

11. **Open-Ended** Draw a quadrilateral on a coordinate grid. Describe a reflection, translation, rotation, and glide reflection. Then draw the image of the quadrilateral for each transformation. **Check students’ work.**

Identify each mapping as a translation, reflection, rotation, or glide reflection. Write the rule for each translation, reflection, rotation, or glide reflection. For glide reflections, write the rule as a composition of a translation and a reflection.

12. $\triangle ABC \rightarrow \triangle DEF$ $T_{<3, -2>}$

13. $\triangle DEF \rightarrow \triangle GHF$ $r_{(180^\circ, F)}$

14. $\triangle DEF \rightarrow \triangle IJK$ $R_y = -1 \circ T_{<8, 0>}$

15. $\triangle GHF \rightarrow \triangle IJK$ $R_x = 3$

$P$ maps to $P'(2, 3)$ by the given glide reflection. What are the coordinates of $P$?

16. $(R_{x-axis} \circ T_{<0, 2>})(P)$ $(2, -5)$

17. $(R_y = x \circ T_{<3, -2>})(P)$ $(0, 4)$

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Find the image of each figure after the transformation $R_m \circ R_\ell$. Is the resulting transformation a translation or a rotation? For a translation, describe the direction and distance. For a rotation, tell the center of rotation and the angle of rotation.

1. To start, if the lines $\ell$ and $m$ are parallel, then it is a __ __ __ translation; directly right; twice the distance between the two lines.

2. rotation; intersection of lines $m$ and $\ell$; $180^\circ$

3. rotation; intersection of lines $m$ and $\ell$; $90^\circ$ clockwise

Graph $\triangle ABC$ and its glide reflection image.

4. $(R_{x\text{-axis}} \circ T_{<2,0>})(\triangle ABC)$
   To start, translate the vertices of $\triangle ABC$ to:
   $A'(3,5), B'(5,1), C'(1,1)$.
   Then, reflect $\triangle A'B'C'$ across __ __ __ $y = 0$

5. $(R_{y\text{-axis}} \circ T_{<0,-3>})(\triangle ABC)$

6. $(R_y = -1 \circ T_{<1,-1>})(\triangle ABC)$
Use the given points and lines. Graph $\overline{XY}$ and its image $\overline{X'Y'}$ after a reflection first across $\ell_1$ and then across $\ell_2$. Is the resulting transformation a translation or a rotation? For a translation, describe the distance and direction. For a rotation, tell the center of rotation and the angle of rotation.

7. $X(4, 3), Y(-2, 1); \ell_1 : y = 2; \ell_2 : x = 2$

8. $X(-3, 4), Y(2, 3); \ell_1 : y = 2; \ell_2 : y = -1$

9. Open-Ended Draw a quadrilateral on a coordinate grid. Draw the image of the quadrilateral for one example of each transformation. Check students' work.
   a. reflection  
   b. translation  
   c. rotation  
   d. glide reflection

Identify each mapping as a translation, reflection, rotation, or glide reflection. Write the rule for each translation, reflection, rotation, or glide reflection. For glide reflections, write the rule as a composition of a translation and a reflection.

10. trapezoid $ABCD \rightarrow$ trapezoid $JI CD$ 
   reflection; $R_{x=-3}$

11. trapezoid $ABCD \rightarrow$ trapezoid $NKLM$ 
   translation; $T<10, 0>$

12. trapezoid $CIJD \rightarrow$ trapezoid $LKNM$ 
   reflection; $R_{x=2}$

13. trapezoid $CIJD \rightarrow$ trapezoid $TSNU$ 
   glide reflection; $R_{x-axis} \circ T<2, 0>$

14. trapezoid $KLMN \rightarrow$ trapezoid $STUN$ 
   rotation; $r(180^\circ, (3, 0))$
Multiple Choice

For Exercises 1–5, choose the correct letter.

1. Which transformation is the same as the composition $R_x = -1 \circ R_x = 2$?  
   [A] $r_{(180^\circ, (-1, 2))}$  
   [B] $r_{(180^\circ, (2, -1))}$  
   [C] $T_{<3, 0>}$  
   [D] $T_{<6, 0>}$  

2. What type of transformation maps $\triangle ABC$ onto $\triangle DEF$?  
   [F] translation  
   [G] rotation  
   [H] reflection  
   [I] glide reflection

3. A triangle is reflected across line $\ell$ and then across line $m$. If the lines intersect, what type of isometry is this composition of reflections?  
   [A] translation  
   [B] rotation  
   [C] reflection  
   [D] glide reflection

4. What type of isometry is shown at the right?  
   [F] translation  
   [H] reflection  
   [G] rotation  
   [I] glide reflection

5. If $(R_y = -1 \circ T_{<0,3>})(X) = X'(3, -2)$, what are the coordinates of $X$?  
   [A] $(-5, -2)$  
   [B] $(-2, -2)$  
   [C] $(-2, -5)$  
   [D] $(3, -3)$

Short Response

6. What type of transformation is shown? Give the translation rule, reflection rule, rotation rule, or composition rule of the glide reflection.

   [2] glide reflection; $R_{y\text{-axis}} \circ T_{<0, -4>}$  
   [1] Student identifies transformation or identifies rule.  
   [0] incorrect or no transformation type given
Compositions of Isometries

You learned that a glide reflection is a composition of a translation and then a reflection. A glide reflection can also be expressed as a combination of three reflections. The first translation step can be accomplished by two reflections, and the final step is a third reflection.

1. Look at the glide reflection at the right. $\triangle ABC$ can be mapped to $\triangle A'B'C'$ by $R_y = 0 \circ T_{6,0}$. It could also be mapped to $\triangle A'B'C'$ by three reflections. What is the glide reflection as the composition of three reflections? Answers may vary. Sample: $R_{x}$-axis $\circ R_x = 2 \circ R_x = -1$

Now consider a rotation and a reflection. In the figure at the right, $r_{(180^\circ,O)}(\text{figure } X) = \text{figure } Y$, and then $R_x = -2(\text{figure } Y) = \text{figure } Z$, so that $(R_x = -2 \circ r_{(180^\circ,O)})(\text{figure } X) = \text{figure } Z$.

All figures are congruent.

2. This rotation and reflection composition can be expressed in many other ways. What is another composition of a reflection and a translation that will map $X$ to $Z$? Answers may vary. Sample: $T_{-4,0} \circ R_{x}$-axis

3. Look at the rotation and reflection composition again. Can it be expressed as a composition of reflections? If so, what is the minimum number of reflections that will map $X$ to $Z$? yes; 3

4. Write a composition of reflection rules that will map $X$ to $Z$. Answers may vary. Sample: $R_x = -5 \circ R_x = -3 \circ R_{x}$-axis

5. Draw a glide reflection on a coordinate grid. Write a composition of a translation and a reflection to describe the glide reflection. Then express this glide reflection as a composition of three reflections. Check students’ work.

6. Draw a rotation and reflection composition on a coordinate grid. Write the composition of a rotation rule and a reflection rule. Then express this composition in two other ways. Check students’ work.
Two congruent figures in a plane can be mapped onto one another by a single reflection, compositions of reflections, or glide reflections.

Compositions of two reflections may be either translations or rotations.

If a figure is reflected across two parallel lines, it is a translation.

If a figure is reflected across intersecting lines, it is a rotation.

The arrow is reflected first across line $\ell$ and then across line $m$. Lines $\ell$ and $m$ are parallel. These two reflections are equivalent to translation of the arrow downward.

The triangle is reflected first across line $\ell$ and then across line $m$. Lines $\ell$ and $m$ intersect at point $X$. These two reflections are equivalent to a rotation. The center of rotation is $X$ and the angle of rotation is twice the angle of intersection, in this case, since lines $\ell$ and $m$ are perpendicular, $2 \times 90^\circ$, or $180^\circ$. Using function notation,

$$(R_m \circ R_\ell)(\triangle ABC) = r_{(180^\circ , X)}(\triangle ABC) = \triangle A'B'C'$$

A composition of a translation and a reflection is a glide reflection.

$\triangle N'O'P'$ is the image of $\triangle NOP$, for the glide reflection $$(R_y = -1 \circ T_{<4, -1>})(\triangle NOP) = \triangle N'O'P'.$$
Reteaching (continued)
Compositions of Isometries

Problem
What transformation maps the figure $ABCD$ onto the figure $EFGH$ shown at the right?

The transformation is a glide reflection. It involves a translation, or glide, followed by a reflection in a line parallel to the direction of the translation.

Exercises
- Draw two pairs of parallel lines that intersect as shown at the right.
- Draw a nonregular quadrilateral in the center of the four lines.
- Use paper folding and tracing to reflect the figure and its images so that there is a figure in each of the nine sections.
- Label the figures 1 through 9 as shown.

Describe a transformation that maps each of the following.

1. figure 4 onto figure 6  \textit{translation}
2. figure 1 onto figure 2  \textit{reflection}
3. figure 7 onto figure 5  \textit{rotation}
4. figure 2 onto figure 9  \textit{glide reflection}
5. figure 1 onto figure 5  \textit{rotation}
6. figure 6 onto figure 7  \textit{glide reflection}
7. figure 8 onto figure 9  \textit{reflection}
8. figure 2 onto figure 8  \textit{translation}

$P(2, 3)$ maps to $P'$ by the given glide reflection. What are the coordinates of $P'$?
(Hint: it may help to graph the transformations.)

9. $R_{x\text{-axis}} \circ T_{<3, -2>} (5, -1)$
10. $R_{y\text{-axis}} \circ T_{<-4, 2>} (2, 5)$
11. $R_{y=x} \circ T_{<0, -3>} (0, 2)$
12. $R_{y=4} \circ T_{<-2, -3>} (0, 8)$
9-5 Additional Vocabulary Support
Congruence Transformations

Use words from the list below to complete the sentences.

<table>
<thead>
<tr>
<th>angle of rotation</th>
<th>center of rotation</th>
<th>congruence transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>congruent</td>
<td>glide reflection</td>
<td>translation</td>
</tr>
<tr>
<td>reflection</td>
<td>rotation</td>
<td>translation</td>
</tr>
</tbody>
</table>

1. In the coordinate plane above, all the triangles are congruent figures.

2. To show that any two figures above are congruent, you can identify a congruence transformation that maps one figure to another.

3. A transformation that maps \( \triangle ABC \) to \( \triangle DEF \) is a translation that slides \( \triangle ABC \) four units to the right and two units down.

4. A transformation that maps \( \triangle DEF \) to \( \triangle GHI \) is a rotation with an angle of rotation of 180° and center of rotation at the origin.

5. A transformation that maps \( \triangle GHI \) to \( \triangle JKL \) is a reflection with line of reflection of \( x = 4 \).

6. A transformation that maps \( \triangle ABC \) to \( \triangle JKL \) is a glide reflection by sliding \( \triangle ABC \) twelve units to the right and two units down and then reflecting across the x-axis.
9-5 Think About a Plan

Congruence Transformations

Use congruence transformations to prove the Isosceles Triangle Theorem.

Given: \( FG \cong FH \)

Prove: \( \angle G \cong \angle H \)

1. Since you want to show that \( \angle G \cong \angle H \), you want to find a congruence transformation that will map \( \angle G \) to \( \angle H \). If the preimage of this transformation is \( \triangle FGH \), what is the image?

\( \triangle FHG \)

2. Identify the congruence transformation as a translation, reflection, rotation, or a composition of rigid motions.

reflection

3. For each rigid motion used in the transformation, describe the translation rule, the line of reflection, or the center and angle of rotation.

The line of reflection is the perpendicular bisector of \( \overline{GH} \).

4. Explain why this transformation maps \( \triangle FGH \) to \( \triangle FHG \).

Let \( M \) be the midpoint of \( \overline{GH} \). Since the line of reflection is the perpendicular bisector, \( \overline{GM} \equiv \overline{HM} \) and the reflection maps \( G \) to \( H \) and \( H \) to \( G \), so it maps \( \overline{GH} \) to \( \overline{HG} \). Because \( \overline{FG} \equiv \overline{FH} \) and distance is preserved under reflections, the reflection maps \( \overline{FG} \) to \( \overline{FH} \).

5. Since this transformation maps point \( F \) to point \( F \), point \( G \) to point \( H \), and point \( H \) to point \( G \), you know that it maps \( \angle G \) to \( \angle H \). Why does it show that \( \angle G \cong \angle H \)?

The transformation maps \( \triangle FHG \) to \( \triangle FGH \). Because a reflection is a rigid motion, it means both distance and angle measure are preserved. \( \angle G \) and \( \angle H \) are congruent because corresponding parts of congruent figures are congruent.
For each coordinate grid, identify a pair of congruent figures. Then determine a congruence transformation that maps the preimage to the congruent image.

1. \( \triangle ABC \cong \triangle GHI \); Answers may vary. Sample: \( r_{(180^\circ, O)}(\triangle ABC) = \triangle GHI \)

2. \( \triangle ABC \cong \triangle DEF \); Answers may vary. Sample: \( T_{-5, -4}(\triangle DEF) = \triangle DEF \)

3. \( \triangle DEF \cong \triangle GHI \); Answers may vary. Sample: \( r_y = 1(\triangle CD) = \triangle EF \)

4. \( \triangle ABC \cong \triangle DEF \); Answers may vary. Sample: \( T_{6, 0}(\triangle ABC) = \triangle IHG \)

5. \( \triangle ABC \cong \triangle DEF \); Answers may vary. Sample: \( T_{-1, 0} \circ r_{y-axis} \circ r_{(90^\circ, O)}(\triangle ABC) = \triangle DEF \)

6. \( \triangle ABC \cong \triangle DEF \); Answers may vary. Sample: \( T_{-2, 0} \circ r_{(180^\circ, O)}(\triangle ABC) = \triangle DEF \)
7. Verify the ASA Postulate for triangle congruence by using congruence transformations.
   **Given:** \( \angle A \cong \angle K, \angle B \cong \angle L, AB \equiv KL \)
   **Prove:** \( \triangle ABC \cong \triangle KLM \)
   Answers may vary. Sample: Translate \( \triangle ABC \) up until points \( B \) and \( L \) coincide. Then reflect across the vertical line containing \( B \) and \( L \).

8. Verify the SSS Postulate for triangle congruence by using congruence transformations.
   **Given:** \( XY \equiv RS, YZ \equiv ST, ZX \equiv TR \)
   **Prove:** \( \triangle XYZ \cong \triangle RST \)
   Answers may vary. Sample: Translate \( \triangle XYZ \) so that points \( Z \) and \( T \) coincide. Then rotate \( \triangle XYZ \) about point \( Z \) until \( YZ \) and \( ST \) coincide.

Determine whether the figures are congruent. If so, describe a congruence transformation that maps one to the other. If not, explain.

9. Not congruent; one figure is smaller than the other figure.

10. Congruent; answers may vary. Sample: draw a line between the two figures and reflect across the line.

11. Error Analysis For the figure at the right, a student says that \( \triangle ABC \cong \triangle DEF \) because \( R_{y-axis}(\triangle ABC) = \triangle DEF \). What is the student’s error?
   The transformation \( R_{y-axis} \) maps \( \triangle ABC \) onto \( \triangle FED \) not \( \triangle DEF \), but \( T_{<4,0>}(\triangle ABC) = \triangle DEF \).

12. Reasoning Suppose one congruence transformation maps \( \triangle LMN \) onto \( \triangle PQR \) and another congruence transformation maps \( \triangle LMN \) onto \( \triangle PRQ \). How would you classify the two triangles? Explain.
   Both \( \triangle LMN \) and \( \triangle PQR \) are isosceles triangles. Since congruence transformations exist, \( \triangle LMN \equiv \triangle PQR \) and \( \triangle LMN \equiv \triangle PRQ \). By the transitive property, \( \triangle PQR \equiv \triangle PRQ \). Then \( PQ \equiv PR \) since they are corresponding parts. By definition, \( \triangle PQR \) is isosceles and so is \( \triangle LMN \).
9-5 Practice

For each coordinate grid, identify a pair of congruent figures. Then determine a congruence transformation that maps the preimage to the congruent image.

1. Compare each pair of figures on the grid.

   Is \( \triangle ABC \cong \triangle DEF \)? \(\text{no}\)

   Is \( \triangle DEF \cong \triangle GHI \)? \(\text{yes}\)

   Is \( \triangle ABC \cong \triangle GHI \)? \(\text{no}\)

Now, determine a congruence transformation that maps one of the congruent triangles to the other.

*Answers may vary.* Sample: \( T_{< -2, -4>}(\triangle DEF) = \triangle GHI \)

2. \( \triangle ABC \cong \triangle DEF \); Answers may vary. Sample: \( T_{< 4, 4>} \circ R_{(90^\circ, 0)}(\triangle ABC) = \triangle LMNP \)

3. \( \triangle LM \cong \triangle OP \); Answers may vary. Sample: \( R_{y-axis} \circ T_{< 1, 0>}(\triangle LM) = \triangle OP \)

Use congruence transformations to verify that \( \triangle ABC \cong \triangle DEF \).

4. \( \triangle ABC \cong \triangle DEF \); Answers may vary. Sample: \( T_{< 0, -2>} \circ R_{y-axis}(\triangle ABC) = \triangle DEF \)

5. \( \triangle ABC \cong \triangle DEF \); Answers may vary. Sample: \( T_{< -1, -4>} \circ r_{(90^\circ, 0)}(\triangle ABC) = \triangle DEF \)
6. Verify the SAS Postulate for triangle congruence by using congruence transformations.

**Given:** \( \angle G \cong \angle M, FG \cong LM, GH \cong MN \)

**Prove:** \( \triangle FGH \cong \triangle LMN \)

Answers may vary. Sample: Reflect \( \triangle FGH \) across the perpendicular bisector of \( GM \) so \( \angle G \) and \( \angle M \) coincide along with congruent side.

7. Verify the ASA Postulate for triangle congruence by using congruence transformations.

**Given:** \( \angle P \cong \angle S, \angle Q \cong \angle T, PQ \cong ST \)

**Prove:** \( \triangle PQR \cong \triangle STU \)

Answers may vary. Sample: Translate \( \triangle PQR \) so that points \( R \) and \( U \) coincide. Then rotate \( \triangle PQR \) about point \( U \) until \( PQ \) and \( ST \) coincide.

Determine whether the figures are congruent. If so, describe a congruence transformation that maps one to the other. If not, explain.

8. Congruent; answers may vary. Sample: draw a line between the two figures and reflect across the line.

9. Not congruent; one figure is smaller than the other figure.

10. To show that \( \triangle ABC \) on the right is an equilateral triangle, what congruence transformation can you use that maps the triangle to itself? Explain.

Answers may vary. Sample: \( r_{60^\circ}, p(\triangle ABC) = BCA \), where \( P \) is a point of concurrency of the triangle. As a result of that transformation and the transitive property, \( AB \cong BC \cong CA \) and \( \angle A \cong \angle B \cong \angle C \).
9-5 Standardized Test Prep
Congruence Transformations

Multiple Choice

For Exercises 1–3, choose the correct letter.

1. Which congruence transformation maps \(\triangle ABC\) to \(\triangle DEF\)?
   - \(A\) \(T_{<5, -5>}\)
   - \(B\) \(r_{(180°, O)}\)
   - \(C\) \(R_{x-\text{axis}} \circ T_{<5, 0>}\)
   - \(D\) \(R_{y-\text{axis}} \circ r_{(90°, O)}\)

2. Which congruence transformation does not map \(\triangle ABC\) to \(\triangle DEF\)?
   - \(E\) \(r_{(180°, O)}\)
   - \(F\) \(R_{x-\text{axis}} \circ R_{y-\text{axis}}\)
   - \(G\) \(T_{<0, 6>} \circ R_{y-\text{axis}}\)
   - \(H\) \(R_{y-\text{axis}} \circ R_{x-\text{axis}}\)

3. Which of the following best describes a congruence transformation that maps \(\triangle FGH\) to \(\triangle LMN\)?
   - \(A\) a reflection only
   - \(B\) a translation only
   - \(C\) a translation followed by a reflection
   - \(D\) a translation followed by a rotation

Short Response

4. What is a congruence transformation that maps \(ABCD\) to \(RSTU\)?
   \(2\) \(T_{<0, -5>} \circ r_{(90°, O)}\)(\(ABCD\)) or equivalent congruence transformation \(1\) gives one correct transformation of a possible composition of transformations that maps \(ABCD\) to \(RSTU\) \(0\) incorrect or no response
Inverse Transformations

When two figures are congruent, there is a congruence transformation that maps the first figure to the second figure and another transformation that maps from the second to the first figure. An inverse transformation of a given congruence transformation is a transformation which maps back to the first figure.

You can use the following rules for translations, reflections, rotations, and compositions to find an inverse transformation given a congruence transformation. For a translation $T_{<h, k>}$, an inverse transformation is $T_{<-h, -k>}$. For a reflection $R_f$, an inverse transformation is the reflection itself $R_f$. For a rotation $r_{(x^\circ, p)}$, an inverse transformation is $r_{(-x^\circ, p)}$ or $r_{(360^\circ - x^\circ, p)}$. For compositions, find the inverse transformation for each transformation of the composition and then compose in reverse order.

Look at the figure at the right. $T_{<-5, -1>}$ maps $\triangle ABC$ to $\triangle DEF$, so $T_{<5, 1>}$ maps $\triangle DEF$ back to $\triangle ABC$. $R_{x\text{-axis}}$ maps $\triangle DEF$ to $\triangle GHI$, so $R_{x\text{-axis}}$ maps $\triangle GHI$ back to $\triangle DEF$. $r_{(90^\circ, O)}$ maps $\triangle GHI$ to $\triangle JKL$, so $r_{(-90^\circ, O)}$ or $r_{(270^\circ, O)}$ maps $\triangle JKL$ back to $\triangle GHI$. The composition $r_{(90^\circ, O)} \circ R_{x\text{-axis}}$ maps $\triangle DEF$ to $\triangle JKL$, so $R_{x\text{-axis}} \circ r_{(270^\circ, O)}$ maps $\triangle JKL$ back to $\triangle DEF$.

For Exercises 1–6, write an inverse transformation that maps $\triangle STU$ to $\triangle PQR$ given the congruence transformation that maps $\triangle PQR$ to $\triangle STU$.

1. $T_{<4, -2>} (\triangle PQR) = \triangle STU$
   $T_{<-4, 2>} (\triangle STU) = \triangle PQR$

2. $R_y = 1 (\triangle PQR) = \triangle STU$
   $R_y = 1 (\triangle STU) = \triangle PQR$

3. $r_{(120^\circ, O)} (\triangle PQR) = \triangle STU$
   $r_{(240^\circ, O)} (\triangle STU) = \triangle PQR$

4. $R_{y\text{-axis}} \circ r_{(210^\circ, O)} (\triangle PQR) = \triangle STU$
   $(r_{(150^\circ, O)} \circ R_{y\text{-axis}}) (\triangle STU) = \triangle PQR$

5. $(T_{<-3, 1>} \circ R_{x\text{-axis}}) (\triangle PQR) = \triangle STU$
   $(R_{x\text{-axis}} \circ T_{<-3, -1>}) (\triangle STU) = \triangle PQR$

6. $(r_{(45^\circ, O)} \circ T_{<-4, -1>}) (\triangle PQR) = \triangle STU$
   $(T_{<4, 1>} \circ r_{(315^\circ, O)}) (\triangle STU) = \triangle PQR$

For Exercises 7–9, use the figure at the right.

7. What is a glide reflection that maps $\triangle ABC$ to $\triangle DEF$? $R_{x\text{-axis}} \circ T_{<5, 0>}$

8. Using the rules described above, what is an inverse transformation of the glide reflection? $T_{<-5, 0>} \circ R_{x\text{-axis}}$

9. If you reverse the order of composition of the transformation in Exercise 8, you get a glide reflection. Does that map $\triangle DEF$ to $\triangle ABC$? Yes
Because rigid motions preserve distance and angle measure, the image of a rigid motion or composition of rigid motions is congruent to the preimage. Congruence can be defined by rigid motions as follows.

Two figures are congruent if and only if there is a sequence of one or more rigid motions that map one figure onto the other.

Because rigid motions map figures to congruent figures, rigid motions and compositions of rigid motions are also called congruence transformations. If two figures are congruent, you can find a congruence transformation that maps one figure to the other.

**Problem**

In the figure at the right, \( \triangle PQR \equiv \triangle STU \). What is a congruence transformation that maps \( \triangle PQR \) to \( \triangle STU \)?

\( \triangle STU \) appears to have the same shape and orientation as \( \triangle PQR \), but rotated \( 90^\circ \), so start by applying the rotation \( r_{(90^\circ, O)} \) on the vertices of \( \triangle PQR \).

\[
r_{(90^\circ, O)}(P) = (-4, 1), \quad r_{(90^\circ, O)}(Q) = (-1, 4), \quad r_{(90^\circ, O)}(R) = (-2, 1)
\]

Graph the image \( r_{(90^\circ, O)}(\triangle PQR) \). A translation of 1 unit to the right and 5 units down maps the image to \( \triangle STU \).

Therefore, \((T_{1, -5} \circ r_{(90^\circ, O)})(\triangle PQR) = \triangle STU\).

**Exercises**

Find a congruence transformation that maps \( \triangle ABC \) to \( \triangle DEF \).

1. Answers may vary. Sample: \((R_{x-axis} \circ T_{<5, 0>})(\triangle ABC) = \triangle DEF\)

2. Answers may vary. Sample: \((r_{(270^\circ, O)} \circ R_{y-axis})(\triangle ABC) = \triangle DEF\)
If you can show that a congruence transformation exists from one figure to another, then you have shown that the figures are congruent.

**Problem**

Verify the SSS Postulate by using a congruence transformation.

**Given:** $JK \equiv RS$, $KL \equiv ST$, $LJ \equiv TR$

**Prove:** $\triangle JKL \equiv \triangle RST$

Start by translating $\triangle JKL$ so that points $J$ and $R$ coincide.

Because you are given that $JK \equiv RS$, there is a rigid motion that maps $JK$ onto $RS$ by rotating $\triangle JKL$ about point $R$ so that $JK$ and $RS$ coincide. Thus, there is a congruence transformation that maps $\triangle JKL$ to $\triangle RST$, so $\triangle JKL \equiv \triangle RST$.

**Exercises**

3. Verify the SAS Postulate for triangle congruence by using congruence transformations.

   **Given:** $\angle R \equiv \angle X$, $RS \equiv XY$, $ST \equiv YZ$

   **Prove:** $\triangle RST \equiv \triangle XYZ$

   Answers may vary. Sample: Since $RS \equiv XY$, translate $\triangle RST$ so $RS$ coincides with $XY$. Then reflect $\triangle RST$ across $XY$ to complete the transformation.

4. Verify the ASA Postulate for triangle congruence by using congruence transformations.

   **Given:** $\angle A \equiv \angle J$, $\angle B \equiv \angle K$, $AB \equiv JK$

   **Prove:** $\triangle ABC \equiv \triangle JKL$

   Answers may vary. Sample: Translate $\triangle ABC$ so that points $C$ and $L$ coincide. Then rotate $\triangle ABC$ about point $L$ until $AB$ and $JK$ coincide.
## 9-6 Additional Vocabulary Support
### Dilations

The column on the left shows the steps used to graph a dilation. Use the column on the left to answer each question in the column on the right.

<table>
<thead>
<tr>
<th><strong>Problem</strong></th>
<th><strong>Graphing a Dilation</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What are the images of the vertices of ( \triangle ABC ) for a dilation centered at the origin with a scale factor of ( n = 2 )? Graph the image of ( \triangle ABC ).</strong></td>
<td><strong>1. Read the example. What do you need to find to solve the problem?</strong></td>
</tr>
<tr>
<td></td>
<td><strong>the coordinates of the vertices for the dilation</strong></td>
</tr>
<tr>
<td>Identify the coordinates of each vertex of ( \triangle ABC ). ( A(0, 3), B(-2, 0), C(2, -2) )</td>
<td><strong>2. The dilation center is the origin. What are the coordinates of the origin?</strong></td>
</tr>
<tr>
<td></td>
<td>(0, 0)</td>
</tr>
<tr>
<td>Use the dilation rule where ( n ) is the scale factor. ( D_n(x, y) = (nx, ny) ) ( D_2(x, y) = (2x, 2y) )</td>
<td><strong>3. What is a dilation rule for a transformation?</strong></td>
</tr>
<tr>
<td></td>
<td><strong>a rule that describes how to map a preimage onto its image</strong></td>
</tr>
<tr>
<td>Find the images of ( A, B, ) and ( C ). ( D_2(A) = (2 \cdot 0, 2 \cdot 3), \text{ or } A'(0, 6) ) ( D_2(B) = (2 \cdot (-2), 2 \cdot 0), \text{ or } B'(-4, 0) ) ( D_2(A) = (2 \cdot 2, 2 \cdot (-2)), \text{ or } C'(4, -4) )</td>
<td><strong>4. How was the rule used to find the images of each vertex?</strong></td>
</tr>
<tr>
<td></td>
<td><strong>The coordinates of the vertex were substituted into the rule to find the coordinates of the image.</strong></td>
</tr>
<tr>
<td>Graph ( A', B', ) and ( C' ). Then draw ( \triangle A'B'C ).</td>
<td><strong>5. What does it mean to graph a point?</strong></td>
</tr>
<tr>
<td></td>
<td><strong>to plot the point on a coordinate grid</strong></td>
</tr>
<tr>
<td></td>
<td><strong>6. Use the grid to the left to graph ( A', B', ) and ( C' ) and draw ( A'B'C' ).</strong></td>
</tr>
</tbody>
</table>
9-6 Think About a Plan
Dilations

Reasoning You are given $\overline{AB}$ and its dilation image $\overline{A'B'}$ with $A, B, A'$, and $B'$ noncollinear. Explain how to find the center of dilation and scale factor.

Know
1. What do you know about the relationship of $A, A'$, and $C$, the center of dilation?
   They are collinear.

2. Is this true also of $B, B'$, and $C$? yes

3. What relationship exists between the lengths of the segments $\overline{A'B'}$ and $\overline{AB}$?
   Their ratio is the scale factor.

Need
4. Is it possible to answer this question using a specific point $C$ and a specific scale factor? Explain.
   No; there is not enough information given. The answer can only be given in general terms.

Plan
5. How do you find the center of dilation?
   Draw a line connecting $A$ and $A'$ and a second line connecting $B$ and $B'$. Point $C$ will be at the point where the two intersect.

6. How do you find the scale factor?
   Simplify the ratio of $\frac{\text{length of image}}{\text{length of preimage}}$ or $\frac{A'B'}{AB}$.
9-6 Practice

Form G

Dilations

The solid-line figure is a dilation of the dashed-line figure. The labeled point is the center of dilation. Tell whether the dilation is an enlargement or a reduction. Then find the scale factor of the dilation.

1. [Diagram 1]
   - Enlargement: \( \frac{5}{3} \)

2. [Diagram 2]
   - Reduction: \( \frac{1}{2} \)

3. [Diagram 3]
   - Enlargement: \( \frac{16}{9} \)

4. [Diagram 4]
   - Enlargement: 4

5. [Diagram 5]
   - Enlargement: 3

6. [Diagram 6]
   - Enlargement: 2

7. [Diagram 7]
   - Enlargement: 2

8. [Diagram 8]
   - Reduction: \( \frac{1}{3} \)

You look at each object described in Exercises 9–11 under a magnifying glass. Find the actual dimension of each object.

9. The image of a ribbon is 10 times the ribbon’s actual size and has a width of 1 cm. 0.1 cm

10. The image of a caterpillar is three times the caterpillar’s actual size and has a width of 4 in. 4 \( \frac{2}{3} \) in.

11. The image of a beetle is five times the beetle’s actual size and has a length of 1.75 cm. 0.35 cm

12. \( \triangle P'Q'R' \) is a dilation image of \( \triangle PQR \). The scale factor for the dilation is 0.12. Is the dilation an enlargement or a reduction? reduction
A dilation has center (0, 0). Find the image of each point for the given scale factor.

13. \(X(3, 4); D_7(X)\) (21, 28)  
14. \(P(-3, 5); D_{1.2}(P)\) (-3.6, 6)  
15. \(Q(0, 4); D_{3.4}(Q)\) (0, 13.6)  
16. \(T(-2, -1); D_4(T)\) (-8, -4)  
17. \(S(5, -6); D_{\frac{25}{3}}(S)\) \((\frac{25}{3}, -10)\)  
18. \(M(2, 2); D_5(M)\) (10, 10)  

19. A square has 16-cm sides. Describe its image for a dilation with center at one of the vertices and scale factor 0.8. The dilation image will be a square with 12.8-cm sides that shares the vertex that is the dilation center with the original square. The sides will be parallel to or along the original sides.

20. Graph pentagon \(ABCDE\) and its image \(A'B'C'D'E'\) for a dilation with center \((0, 0)\) and a scale factor of 1.5. The vertices of \(ABCDE\) are: \(A(0, 3), B(3, 3), C(3, 0), D(0, -3), E(-1, 0)\).

Copy \(\triangle BCD\) and point \(X\) for each of Exercises 21–23. Draw the dilation image \(\triangle B'C'D'\).

21. \(D_{(1.5, X)}(\triangle BCD)\)

22. \(D_{(1.5, B)}(\triangle BCD)\)

23. \(D_{(0.8, C)}(\triangle BCD)\)
9-6 Practice

Dilations

The dashed-line figure is a dilation image of the solid-line figure. The labeled point is the center of dilation. Tell whether the dilation is an enlargement or a reduction. Then find the scale factor of the dilation.

1. To start, identify whether the image is larger or smaller than the preimage.
Next, find the scale factor:
\[
\frac{\text{image length}}{\text{preimage length}} = \frac{1}{2} \text{ reduction}
\]

2. 
3. 

4. 

5. 

A dilation has center (0, 0). Find the image of each point for the given scale factor.

6. \( B(2, 3); D_6(B) \) 
   \( (12, 18) \)
   \( D_6(B) = (2 \cdot \frac{6}{3}, 3 \cdot \frac{6}{3}) = (12, 18) \)

7. \( C(7, 2); D_3(C) \) 
   \( (21, 6) \)

8. \( U(-3, -2); D_3(U) \) 
   \( (-9, -6) \)

9. \( A(0, -5); D_{\frac{5}{3}}(A) \) 
   \( (0, -2) \)

10. \( G(3, 1); D_{-4}(G) \) 
    \( (-12, -4) \)

11. \( R(-3, 5); D_{\frac{3}{4}}(R) \) 
    \( (-\frac{3}{4}, 1\frac{1}{4}) \)

12. \( X(\frac{1}{2}, -3); D_{-2}(X) \) 
    \( (-1, 6) \)
You look at each object described in Exercises 13–15 under a magnifying glass. Find the actual dimension of each object.

13. The image of a bug is 5 times its actual size and has a width of 1.5 cm.
   \[ \text{image length} = \text{scale factor} \cdot \text{actual length} \]
   \[ ? = ? \cdot 1.5 \, \text{cm}; 5; x; 0.3 \, \text{cm} \]

14. The image of a worm is 4 times its actual size and has a length of 7 cm.
   \[ \frac{7}{4} \, \text{cm} \]

15. The image of a hair is 10 times its actual size and has a length of 0.4 cm.
   \[ 0.04 \, \text{cm} \]

16. A dilation maps \( \triangle QRS \) to \( \triangle Q'R'S' \). \( QR = 10 \, \text{in.} \) and \( Q'R' = 12 \, \text{in.} \)
   If \( RS = 12 \, \text{in.} \), what is \( R'S' \)?
   \[ 14.4 \, \text{in.} \]

17. A dilation on a coordinate grid has center \((0, 0)\) and scale factor 2.5. Point \( A \) is at \((3, 7)\). What is the \( y \)-coordinate of the image of \( A \)?
   \[ 17.5 \]

18. \( \triangle A'B'C' \) is a dilation image of \( \triangle ABC \). The scale factor for the dilation is 1.25.
   Is the dilation an enlargement or a reduction?
   Enlargement

19. A square has 12-cm sides. Describe its image for a dilation with center at one of the vertices and scale factor 0.4.
   The dilation image will be a square with 4.8-cm sides that shares the vertex that is the dilation center with the original square. The sides will be parallel to or along the original sides.

20. Graph pentagon \( PENTA \) and its image
   \[ D_{0.5}(PENTA) = P'E'N'T'A'. \]
   The vertices of \( PENTA \) are: \( P(0, 4), E(6, 6), N(4, 0), T(0, -4), A(-2, 0) \).

A dilation maps \( \triangle MNO \) onto \( \triangle M'N'O' \). Find the missing values.

21. \( MN = 2 \, \text{in.}, M'N' = 3.5 \, \text{in.} \)
   \( NO = 3 \, \text{in.}, N'O' = \boxed{5.25} \, \text{in.} \)
   \( MO = 4 \, \text{in.}, M'O' = \boxed{7.0} \, \text{in.} \)

22. \( MN = 2 \, \text{cm}, M'N' = 1.6 \, \text{cm} \)
   \( NO = 5 \, \text{cm}, N'O' = \boxed{4.0} \, \text{cm} \)
   \( MO = 6 \, \text{cm}, M'O' = \boxed{4.8} \, \text{cm} \)
9-6 Standardized Test Prep
Dilations

Gridded Response

The solid-line figure is a dilation of the dashed-line figure. The labeled point is the center of dilation. Find the scale factor for each dilation. Use whole numbers or decimals. Enter your responses on the grid provided.

1. 

2. 

Solve the problem and enter your response on the grid provided.

3. The image of an eraser in a magnifying glass is three times the eraser’s actual size and has a width of 14.4 cm. What is the actual width in cm?

4. A square on a transparency is 1.7 in. long. The square’s image on the screen is 11.05 in. long. What is the scale factor of the dilation?

5. A dilation maps $\triangle LMN$ to $\triangle L’M’N’$. $MN = 14$ in. and $M’N’ = 9.8$ in.
If $LN = 13$ in., what is $L’N’$?

Answers

1. 

2. 

3. 

4. 

5. 

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9-6 Enrichment
Dilations

Three-Dimensional Dilations

Refer to the cube in Figure 1 and its dilation in Figure 2 to complete the following exercises.

1. Give the coordinates of all vertices of the preimage. Remember, each point in the three-dimensional coordinate system has coordinates in the form \((x, y, z)\).
   - a. \(A (2, 0, 2)\)
   - b. \(B (0, 0, 2)\)
   - c. \(C (0, 2, 2)\)
   - d. \(D (2, 2, 2)\)
   - e. \(E (2, 0, 0)\)
   - f. \(F (0, 0, 0)\)
   - g. \(G (0, 2, 0)\)
   - h. \(H (2, 2, 0)\)

2. Give the coordinates of all vertices of the image.
   - a. \(A' (4, 0, 4)\)
   - b. \(B' (0, 0, 4)\)
   - c. \(C' (0, 4, 4)\)
   - d. \(D' (4, 4, 4)\)
   - e. \(E' (4, 0, 0)\)
   - f. \(F' (0, 0, 0)\)
   - g. \(G' (0, 4, 0)\)
   - h. \(H' (4, 4, 0)\)

3. What is the center of the dilation? \((0, 0, 0)\)

4. What is the scale factor? \(2\)

5. How do the lengths of the edges of the cubes compare?
   
   Each edge in the image is double that in the preimage.

6. How do the faces of the cubes compare?
   - Samples: Three pairs of faces are coplanar for image and preimage; three pairs of faces are parallel; each face in the image has two times the perimeter and four times the area of the corresponding face in the preimage.

7. How do the total surface areas of the cubes compare?
   - Surface area of image = 96 sq. units; surface area of preimage = 24 sq. units; the ratio of the surface areas is \(4 : 1\).

8. How do the volumes of the cubes compare?
   - Volume of image = 64 cubic units; volume of preimage = 8 cubic units; the ratio of the volumes is \(8 : 1\).

9. How do the dilations in three dimensions appear to compare with dilations in two dimensions?
   - Dilations in three dimensions also scale each dimension by a common scale factor, like dilations in two dimensions.
9-6  
Reteaching
Dilations

A dilation is a transformation in which a figure changes size. The preimage and image of a dilation are similar. The scale factor of the dilation is the same as the scale factor of these similar figures.

To find the scale factor, use the ratio of lengths of corresponding sides. If the scale factor of a dilation is greater than 1, the dilation is an enlargement. If it is less than 1, the dilation is a reduction.

Using function notation, a dilation with center C and scale factor \( n > 0 \) can be written as \( D(n, C) \). If the dilation is in the coordinate plane with center at the origin, the dilation with scale factor \( n \) can be written as \( D_n \). For a point \( P(x, y) \), the image is \( D_n(P) = (nx, ny) \).

**Problem**

\( \triangle X'Y'Z' \) is the dilation image of \( \triangle XYZ \). The center of dilation is \( X \). The image of the center is itself, so \( X' = X \).

The scale factor, \( n \), is the ratio of the lengths of corresponding sides.

\[
 n = \frac{X'Z'}{XZ} = \frac{30}{12} = \frac{5}{2}
\]

This dilation is an enlargement with a scale factor of \( \frac{5}{2} \).

**Exercises**

For each of the dilations below, \( A \) is the center of dilation. Tell whether the dilation is a reduction or an enlargement. Then find the scale factor of the dilation.

1. \( A = A' \)  
   \[ \begin{array}{c}
   B \quad 8 \\
   B' \quad 10 \\
   C \quad C' \\
   \end{array} \]  
   enlargement; \( \frac{5}{4} \)

2. \( A = A' \)  
   \[ \begin{array}{c}
   B' \quad 8 \\
   B \quad 10 \\
   C' \quad C \\
   \end{array} \]  
   reduction; \( \frac{4}{5} \)

3. \( AB = 2; A'B' = 3 \)  
   enlargement; \( \frac{3}{2} \)

4. \( DE = 3; D'E' = 6 \)  
   enlargement; \( 2 \)
Problem

Quadrilateral $ABCD$ has vertices $A(-2, 0), B(0, 2), C(2, 0),$ and $D(0, -2)$. Find the coordinates of the vertices of $D_2(ABCD)$. Then graph $ABCD$ and its image $D_2(ABCD)$.

To find the image of the vertices of $ABCD$, multiply the $x$-coordinates and $y$-coordinates by 2.

$D_2(A) = (2 \cdot (-2), 2 \cdot 0) = A’(-4, 0)$

$D_2(B) = (2 \cdot 0, 2 \cdot 2) = B’(0, 4)$

$D_2(C) = (2 \cdot 2, 2 \cdot 0) = C’(4, 0)$

$D_2(D) = (2 \cdot 0, 2 \cdot (-2)) = D’(0, -4)$

Exercises

Use graph paper to complete Exercise 5.

5. a. Draw a quadrilateral in the coordinate plane. Check students’ work.

b. Draw the image of the quadrilateral for dilations centered at the origin with scale factors $\frac{1}{2}, 2,$ and 4.

Graph the image of each figure for a dilation centered at the origin with the given scale factor. Check students’ graphs.
9-7

Additional Vocabulary Support

Similarity Transformations

Is there a similarity transformation that maps ΔABC to ΔRST? If so, identify the similarity transformation. Use words from the list below to complete the sentences and answer the question.

- composition
- dilation
- reflection
- scale factor
- similarity transformation
- line of reflection
- translation
- similar
- reflection
- line of reflection
- translation
- composition
- dilation
- scale factor
- reflection
- line of reflection
- translation
- similar

1. ΔABC and ΔRST appear to be __________ similar figures.

2. Two figures are similar if and only if there is a __________ that maps one figure to another.

3. A similarity transformation is a __________ of rigid motions and dilations.

4. Since ΔRST is an enlargement of ΔABC, perform a __________ dilation with center at the origin and a __________ scale factor of 2.

5. Since the resulting image and ΔRST have opposite orientations, perform a __________ reflection across the x-axis, which is the __________ line of reflection.

6. Finally, apply a __________ that slides the second image seven units to the left to ΔRST.
A printing company enlarges a banner for a graduation party by a scale factor of 8.

a. What are the dimensions of the larger banner?

b. How can the printing company be sure that the enlarged banner is similar to the original? Explain.

1. What is the scale factor for this problem?
   8

2. What can you do with the scale factor to find the dimensions of the large banner?
   Multiply the dimensions of the original banner by the scale factor.

3. What is the width of the larger banner?
   104 in.

4. What is the height of the larger banner?
   24 in.

5. What kind of transformation did the printing company apply to the original banner in order to produce the larger banner?
   dilation

6. Explain how the printing company can be sure that the enlarged banner is similar to the original.
   Answers may vary. Sample: Dilation is a similarity transformation so the image (the larger banner) should be similar to the original banner.
9-7 Practice Form G

Similarity Transformations

ΔABC has vertices A(−2, 2), B(2, 0), and C(1, −2). For each similarity transformation, draw the image.

1. \(D_2 \circ R_{y\text{-axis}}\)

2. \(r_{90^\circ} \circ D_{1.5}\)

3. \(D_{0.5} \circ T_{<2, -2>}\)

4. \(r_{270^\circ} \circ D_2 \circ R_{x\text{-axis}}\)

For each graph, describe the composition of transformations that maps \(\triangle DEF\) to \(\triangle LMN\).

5. Answers may vary. Sample: \((R_{x\text{-axis}} \circ D_2)(\triangle DEF) = \triangle LMN\)

6. Answers may vary. Sample: \((D_{0.5} \circ r_{(180^\circ, 0)})(\triangle DEF) = \triangle LMN\)
For each pair of figures, determine if there is a similarity transformation that maps one figure onto the other. If so, identify the similarity transformation and write a similarity statement. If not, explain.

7. Not similar; the ratio of the corresponding longer sides is not the same as the ratio of the corresponding shorter sides.

8. Similar; answers may vary. Sample: translate $\triangle GHI$ so that vertices $I$ and $L$ coincide. Then rotate $\triangle GHI$ 270° about the point $L$, and dilate with a scale factor of $\frac{5}{3}$ and center at $L$; $\triangle GHI \sim \triangle JKL$

Determine whether or not each pair of figures below are similar. Explain your reasoning.

9. Similar; answers may vary. Sample: reflect the smaller figure across a vertical line, translate the small figure so that their tips meet and then enlarge by some scale factor until the figures coincide.

10. Not similar; the sides are not in proportion.

11. **Reasoning** A transformation maps each point $(x, y)$ of $\triangle ABC$ to the point $(-4x + 3, -4y + 2)$. Is $\triangle ABC$ similar to the image of $\triangle ABC$? Explain.

   Yes; the transformation is equivalent to the similarity transformation $T_{<3,2>} \circ r_{(180°, O)} \circ D_4$, since $(T_{<3,2>} \circ r_{(180°, O)} \circ D_4)(x, y) = (T_{<3,2>} \circ r_{(180°, O)})(4x, 4y) = (T_{<3,2>})(-4x, -4y) = (-4x + 3, -4y + 2)$

12. **Reasoning** Do the compositions $T_{<4,-2>} \circ D_2$ and $D_2 \circ T_{<2,-1>}$ describe the same similarity transformation? Explain.

   Yes; for each point $(x, y)$, $(T_{<4,-2>} \circ D_2)(x, y) = T_{<4,-2>}(2x, 2y) = (2x + 4, 2y - 2)$ and $(D_2 \circ T_{<2,-1>})(x, y) = D_2(x + 2, y - 1) = (2(x + 2), 2(y - 1)) = (2x + 4, 2y - 2)$, so both transformations map each point $(x, y)$ to $(2x + 4, 2y - 2)$. 

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\( \triangle LMN \) has vertices \( L(0, 2), M(2, 2), \) and \( N(0, 1) \). For each similarity transformation, draw the image.

1. \( D_2 \circ R_{x-\text{axis}} \)
   Find the image of each vertex.
   \[
   (D_2 \circ R_{x-\text{axis}})(L) = D_2(0, -2) = (0, -4)
   \]
   \[
   (D_2 \circ R_{x-\text{axis}})(M) = D_2(2, -2) = (4, -4)
   \]
   \[
   (D_2 \circ R_{x-\text{axis}})(N) = D_2(0, -1) = (0, -2)
   \]
   Then plot the points and draw the image.

2. \( D_{0.5} \circ r_{(90^\circ, o)} \)

3. \( T_{<-3, -2>} \circ D_{1.5} \)

For each graph, describe the composition of transformations that maps \( \triangle ABC \) to \( \triangle FGH \).

4. \( (D_{1.5} \circ R_{y-\text{axis}})(\triangle ABC) = \triangle FGH \)

5. \( (r_{(90^\circ, o)} \circ D_{0.5})(\triangle ABC) = \triangle FGH \)
For each pair of figures, determine if there is a similarity transformation that maps one figure onto the other. If so, identify the similarity transformation and write a similarity statement. If not, explain.

6. Similar; answers may vary. Sample: rotate the larger rectangle by 90°, dilate with a scale factor of \( \frac{2}{5} \), and then translate so vertices \( A \) and \( M \) coincide; \( ABCD \sim MNPQ \)

7. Similar; answers may vary. Sample: rotate \( \triangle LMN \) 180° so that \( \angle JLK \) and \( \angle MLN \) coincide. Then dilate with some scale factor of \( k \) and center at \( L \) so that \( MN \) and \( JK \) coincide; \( \triangle LMN \sim \triangle JKL \)

Determine whether or not each pair of figures below are similar. Explain your reasoning.

8. Not similar; answers may vary. Sample: the figures have the same height but different widths so corresponding lengths are not in proportion.

9. Similar; answers may vary. Sample: dilate the smaller figure by some scale factor so it has the same size as the larger figure, and then translate onto the larger figure.

10. **Reasoning** A similarity transformation is described by the composition \( T_{<2,1>} \circ D_3 \). If the order of the composition is changed to be \( D_3 \circ T_{<2,1>} \), does that describe the same transformation? Explain.

   No; for each point \((x, y), (T_{<2,1>} \circ D_3)(x, y) = T_{<2,1>} D_3(x, y) = (3x + 2, 3y + 1), \) while \((D_3 \circ T_{<2,1>})(x, y) = D_3(x + 2, y + 1) = (3(x + 2), 3(y + 1)) = (3x + 6, 3y + 3), \) so the two compositions do not describe the same transformation.
Multiple Choice

For Exercises 1–3, choose the correct letter.

1. Which similarity transformation maps \(\triangle ABC\) to \(\triangle DEF\)?
   - A: \(R_{x\text{-axis}} \circ D_{0.5}\)
   - B: \(r_{(270^\circ, O)} \circ D_{0.5}\)
   - C: \(R_{x\text{-axis}} \circ D_2\)
   - D: \(r_{(90^\circ, O)} \circ D_2\)

2. Which similarity transformation does not map \(\triangle PQR\) to \(\triangle STU\)?
   - F: \(r_{(180^\circ, O)} \circ D_2\)
   - G: \(D_2 \circ r_{(180^\circ, O)}\)
   - H: \(D_2 \circ r_{x\text{-axis}} \circ r_{y\text{-axis}}\)
   - I: \(D_2 \circ r_{x\text{-axis}} \circ r_{(90^\circ, O)}\)

3. Which of the following best describes a similarity transformation that maps \(\triangle JKP\) to \(\triangle LMP\)?
   - B: a rotation followed by a dilation
   - A: a dilation only
   - C: a reflection followed by a dilation
   - D: a translation followed by a dilation

Short Response

4. \(\triangle ABC\) has vertices \(A(1, 0), B(2, 4),\) and \(C(3, 2)\). \(\triangle RST\) has vertices \(R(0, 3), S(-12, 6),\) and \(T(-6, 9)\). What is a similarity transformation that maps \(\triangle ABC\) to \(\triangle RST\)?
   
   \[2] (D_3 \circ r_{(90^\circ, O)})(\triangle ABC)\] or equivalent similarity transformation \[1\] gives one correct transformation of a possible composition of transformations that maps \(\triangle ABC\) to \(\triangle RST\) \[0\] incorrect or no response
Horizontal and Vertical Stretches

In the coordinate plane, a horizontal stretch with center at the origin and scale factor $n$ maps a point $(x, y)$ to the point $(nx, y)$. Likewise, a vertical stretch with center at the origin and scale factor $n$ maps a point $(x, y)$ to the point $(x, ny)$.

Under horizontal and vertical stretches, images are neither congruent nor similar to the preimage, but certain classifications of figures can be preserved.

For Exercises 1–3, $\triangle ABC$ is the isosceles triangle with vertices $A(-a, 0), B(0, b)$, and $C(a, 0)$.

1. What are the coordinates of the vertices of the triangle after a horizontal stretch with scale factor $n$? $A'(−na, 0), B'(0, b), C'(na, 0)$

2. What are the coordinates of the vertices of the triangle after a vertical stretch with scale factor $n$? $A'(-a, 0), B'(0, nb), C'(a, 0)$

3. Is the image of $\triangle ABC$ after a horizontal stretch an isosceles triangle? after a vertical stretch? yes; yes

4. Suppose the vertices of isosceles triangle $\triangle ABC$ are $A(0, 0), B(a, b)$, and $C(2a, 0)$. Are the images of $\triangle ABC$ after horizontal or vertical stretches isosceles triangles? yes

For Exercises 5–8, $ABCD$ is a square with vertices $A(-a, 0), B(0, a), C(a, 0)$, and $D(0, -a)$.

5. What are the coordinates of the vertices of $ABCD$ after a horizontal stretch with scale factor $n$? $A'(-na, 0), B'(0, a), C'(na, 0), D'(0, -a)$

6. What are the coordinates of the vertices of $ABCD$ after a vertical stretch with scale factor $n$? $A'(-a, 0), B'(0, na), C'(a, 0), D'(0, -na)$

7. Is the image of $ABCD$ after a horizontal stretch a square? after a vertical stretch? no; no

8. What type of quadrilateral is the image of $ABCD$ after a horizontal and vertical stretch? rhombus

9. Suppose the vertices of square $ABCD$ are $A(0, a), B(a, a), C(a, 0)$, and $D(0, 0)$. What type of quadrilateral is the image of $ABCD$ after a horizontal or vertical stretch? rectangle
9-7 Reteaching
Similarity Transformations

Dilations and compositions of dilations and rigid motions form a special class of transformations called similarity transformations. Similarity can be defined by similarity transformations as follows.

Two figures are similar if and only if there is a similarity transformation that maps one figure to the other.

Thus, if you have a similarity transformation, the image of the transformation is similar to the preimage. Use the rules for each transformation in the composition to find the image of a point in the transformation.

Problem

\( \triangle ABC \) has vertices \( A(-1, 1), B(1, 3), \) and \( C(2, 0). \) What is the image of \( \triangle ABC \) when you apply the similarity transformation \( T_{-2, -5} \circ D_3? \)

For any point \((x, y), D_3(x, y) = (3x, 3y)\) and \( T_{-2, -5}(x, y) = (-2x - 5) \).

\[
(T_{-2, -5} \circ D_3)(A) = T_{-2, -5}(D_3(A)) = T_{-2, -5}((3x, 3y)) = (-2x - 5, -3y - 15)
\]

\[
(T_{-2, -5} \circ D_3)(B) = T_{-2, -5}((3x, 3y)) = (-2x - 5, -3y - 15)
\]

\[
(T_{-2, -5} \circ D_3)(C) = T_{-2, -5}((3x, 3y)) = (-2x - 5, -3y - 15)
\]

Thus, the image has vertices \( A'(-5, 2), B'(1, 4), \) and \( C'(4, -5) \) and is similar to \( \triangle ABC. \)

Exercises

\( \triangle ABC \) has vertices \( A(-2, 1), B(-1, -2), \) and \( C(2, 2). \) For each similarity transformation, draw the image.

1. \( D_2 \circ R_{y-axis} \)
2. \( r_{90^\circ, O} \circ D_2 \)
If you can show that a similarity transformation maps one figure to another, then you have shown that the two figures are similar.

**Problem**

Show that $\triangle JKL \sim \triangle RST$ by finding a similarity transformation that maps one triangle to the other.

$\triangle RST$ appears to be twice the size of $\triangle JKL$, so dilate by scale factor of 2. Map each vertex using $D_2$.

$D_2(J) = (0, 4) = J'$

$D_2(K) = (4, 2) = K'$

$D_2(L) = (2, 0) = L'$

Graph the image of the dilation $\triangle J'K'L'$.

$\triangle J'K'L'$ is congruent to $\triangle RST$ and can be mapped to $\triangle RST$ by the glide reflection $R_{x-axis} \circ T_{< -5, 0>} \circ D_2$.

Verify that each vertex of $\triangle J'K'L'$ maps to a vertex of $\triangle RST$.

$(R_{x-axis} \circ T_{< -5, 0>})(J') = R_{x-axis}(-5, 4) = (-5, -4) = R$

$(R_{x-axis} \circ T_{< -5, 0>})(K') = R_{x-axis}(-1, 2) = (-1, -2) = S$

$(R_{x-axis} \circ T_{< -5, 0>})(L') = R_{x-axis}(-3, 0) = (-3, -0) = T$

Thus, the similarity transformation $R_{x-axis} \circ T_{< -5, 0>} \circ D_2$ maps $\triangle JKL$ to $\triangle RST$.

**Exercises**

For each pair of figures, find a similarity transformation that maps $\triangle ABC$ to $\triangle FGH$. Then, write the similarity statement.

3. Answers may vary. Sample: $(T_{<-4, -10>} \circ D_3)(\triangle ABC) = \triangle FGH$

4. Answers may vary. Sample: $(r_{(270^\circ, 0)} \circ D_{1.5})(\triangle ABC) = \triangle FGH$
Chapter 9 Quiz 1  
Lessons 9-1 through 9-4

Do you know HOW?

Tell whether the transformation appears to be a rigid motion. Explain.

1. [Diagram of preimage and image]
   - Yes; preserves distance and angle measure

2. [Diagram of preimage and image]
   - No; does not preserve distance

3. If a transformation maps \( GHIJ \) to \( G'H'I'J' \), what is the image of \( I' \)? What is the image of \( G'H' \)?

4. Point \( R(x, y) \) moves 13 units right and 14 units down. What is a rule that describes this translation? \( T_{13, -14}(R) \)

Draw the image of each figure for the given transformation.

5. \( R_{x-axis}(ABCD) \)

6. \( r_{(90^\circ, O)}(ABCD) \)

7. \( \triangle ONM \) has vertices \( O(-4, 2), N(3, 6), \) and \( M(0, 3) \). What are the coordinates of the vertices of \( (R_y = 1 \circ T_{<3, 0>})(\triangle ONM) \) \( (-1, 0), (6, -4), (3, -1) \)

8. Is the transformation of \( \overline{AB} \) with vertices \( A(2, 3) \) and \( B(-1, 2) \), first across \( x = 4 \), and then across \( y = -2 \), a translation or a rotation? For a translation, describe the direction and distance. For a rotation, tell the center of rotation and the angle of rotation. \( \text{rotation around (4, -2); 180^\circ} \)

Do you UNDERSTAND?

9. **Reasoning** The point \( (-1, -1) \) is the image under the translation \( T_{<-5, 5>}(x, y) \). What is its preimage? \( (4, -6) \)

10. **Reasoning** In the diagram, is one figure a reflection image, a translation image, or a rotation image of the other? **rotation image**
Chapter 9 Quiz 2  
Lessons 9-5 through 9-7  

Do you know HOW?  

Write a congruence or similarity statement for the two figures in each coordinate grid. Then write a congruence transformation or similarity transformation that maps one figure to the other.

1. \( \triangle ABC \cong \triangle GHI \); Answers may vary.  
   Sample: \((R_x\text{-axis} \circ T_{<3, -1>})(\triangle ABC) = \triangle GHI\)

2. \( \triangle DEF \cong \triangle PQR \); Answers may vary.  
   Sample: \((T_{<1, -3>} \circ r_{(90^\circ, 0)})(\triangle DEF) = \triangle PQR\)

3. \( \triangle LMN \sim \triangle STU \); Answers may vary.  
   Sample: \((T_{<4, -4>} \circ D_{0,5})(\triangle LMN) = \triangle STU\)

4. \( \triangle ABC \sim \triangle LNM \); Answers may vary.  
   Sample: \((R_x\text{-axis} \circ D_2)(\triangle ABC) = \triangle LNM\)

5. The solid-line figure is a dilation of the dashed-line figure with center of dilation \(M\). Is the dilation an enlargement or a reduction?  
   What is the scale factor of the dilation? \(\text{enlargement; } \frac{10}{7}\)

6. Given \(P(3, 11)\), what are the coordinates of \(D_3(P)\)? \((9, 33)\)

7. Given \(P(2, -4)\), what are the coordinates of \(D_4(P)\)? \((8, -16)\)

Do you UNDERSTAND?  

8. Compare and Contrast  Explain how a similarity transformation is the same as or different from a congruence transformation. In a similarity transformation, the image and the preimage are similar but not usually congruent. In a congruence transformation, the preimage and image are congruent. Similarity transformations are compositions of rigid motions and dilations, while congruence transformations are compositions of rigid motions.

9. Error Analysis  A student used image and preimage lengths of an enlargement to find a scale factor of \(\frac{3}{4}\). Explain her possible error.  
   An enlargement must have a scale factor greater than 1. The image length should be the numerator and the preimage length should be the denominator.
Chapter 9 Test

Do you know HOW?

1. \( \triangle R'S'T' \) is a translation image of \( \triangle RST \). What is a rule for the translation?
   \[ T_{< -7, 3>} (x, y) \]

2. Is a glide reflection a rigid motion? Explain. Yes; a glide reflection preserves distance and angle measure because it is a composition of translation and reflection.

Find the coordinates of the vertices of each image.

3. \( R_y(\text{axis})(QRST) \)
   \[ Q'(-1, 5), R'(-3, -1), S'(0, 0), T'(2, 3) \]

4. \( r_{(270^\circ, 0)}(QRST) \)
   \[ Q'(5, -1), R'(-1, -3), S'(0, 0), T'(3, 2) \]

5. \( D_5(QRST) \)
   \[ Q'(5, 25), R'(15, -5), S'(0, 0), T'(-10, 15) \]

6. \( T_{<2, -5>}(QRST) \)
   \[ Q'(3, 0), R'(5, -6), S'(2, -5), T'(0, -2) \]

7. \( (R_y = -2 \circ T_{< -4, 0>})(QRST) \)
   \[ Q'(-3, -9), R'(-1, -3), S'(-4, -4), T'(-6, -7) \]

Write a single transformation rule that has the same effect as each composition of transformations.

8. \( T_{<6, 7>} \circ T_{< -10, 7>} \)
   \[ T_{< -4, 14>} \]

9. \( R_x = 6 \circ R_x = 1 \)
   \[ T_{<10, 0>} \]

10. \( R_y = 2 \circ T_{<0, 1>} \)
    \[ R_y = 1.5 \]

Identify the rigid motion that maps the solid-line figure onto the dotted-line figure.

11.translation

12.reflection

Find the image of each point for the given dilation.

13. \( L(6, 4); D_{1.2}(L) \)
    \[ (7.2, 4.8) \]

14. \( S(-2, -6); D_{0.25}(S) \)
    \[ (-0.5, -1.5) \]

15. \( W(-3, 2); D_5(W) \)
    \[ (-15, 10) \]
Chapter 9 Test (continued)  Form G

Write a congruence or similarity statement for the two figures in each coordinate grid. Then write a congruence transformation or similarity transformation that maps one figure to the other.

16. \( \triangle ABC \cong \triangle PQR; \) Answers may vary.
Sample: \( (r_{90^\circ}, O) \circ R_y \triangle ABC = \triangle PQR \)

17. \( \triangle LMN \sim \triangle UVW; \) Answers may vary.
Sample: \( (r_{180^\circ}, O) \circ D_2 \triangle LMN = \triangle UVW \)

18. \( \triangle GHI = \triangle JKL; \) Answers may vary.
Sample: \( (R_x \circ r_{270^\circ}, O) \triangle GHI = \triangle JKL \)

19. \( \triangle FGH \cong \triangle KLM; \) Answers may vary.
Sample: \( (T_{<1,-5>}, r_{y-axis}) \triangle FGH = \triangle KLM \)

Do you UNDERSTAND?

20. Reasoning  Lines \( a \) and \( b \) are parallel. The letter \( A \) is reflected first across line \( a \) so that the image is between lines \( a \) and \( b \). The figure is then reflected across line \( b \). Describe the image of \( A \).  This series of reflections is a translation. The image of \( A \) is the same size and in the same orientation as the original, but it is translated twice the distance between lines \( a \) and \( b \).

21. Vocabulary  What is the image of \( (-6, 6) \) after a reflection across the \( y \)-axis?
\( (6, 6) \)

22. Coordinate Geometry  \( \triangle ABC \) has vertices at \( A(0, 5), \ B(4, 4), \) and \( C(-1, 0) \). Graph the image \( T_{<-1,-2>} \triangle ABC \).

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Chapter 9 Quiz 1

Lessons 9-1 through 9-4

Do you know HOW?

Tell whether the transformation appears to be a rigid motion. Explain.

1. yes; preserves distance and angle measure
2. no; does not preserve distance

3. If a transformation maps \( \triangle ABC \) to \( \triangle A'B'C' \), what is the image of \( A \)? \( A' \)

4. If a transformation maps \( \triangle ABC \) to \( \triangle A'B'C' \), what is the image of \( BC \)? \( B'C' \)

5. Point \( R(x, y) \) moves 7 units left and 3 units up. What is a rule that describes this translation? \( T_{-7, 3}(R) \)

Draw the image of each figure for the given transformation.

6. \( T_{3, 2}(\triangle ABC) \)

7. \( R_y(\triangle ABC) \)

8. \( \triangle XYZ \) has vertices \( X(-2, 2), Y(-1, 5), \) and \( Z(2, 3) \). What are the coordinates of the vertices of \( (R_x=3 \circ T_{<0, -3>})(\triangle XYZ) \)? \( (8, -1), (7, 2), (4, 0) \)

Do you UNDERSTAND?

9. Reasoning The point \( (1, 1) \) is the image under the translation \( T_{<3, -3>} \). What is the preimage of this point? \( (-2, 4) \)

10. Reasoning In the diagram, is one figure a reflection image, a translation image, or a rotation image of the other? rotation image
Chapter 9 Quiz 2  
Lessons 9-5 through 9-7

Do you know HOW?

Write a congruence or similarity statement for the two figures in each coordinate grid. Then write a congruence transformation or similarity transformation that maps one figure to the other.

1. \[ \triangle ABC \cong \triangle FGH; \text{ Answers may vary.} \]
   Sample: \((R_{x-axis} \circ T_{<5, 0>})(\triangle ABC) = \triangle FGH\)

2. \[ \triangle JKL \cong \triangle PQR; \text{ Answers may vary.} \]
   Sample: \((T_{<3, 0>} \circ r_{(90^\circ, O)})(\triangle JKL) = \triangle PQR\)

3. \[ \triangle DEF \sim \triangle STU; \text{ Answers may vary.} \]
   Sample: \((T_{<0, -4>} \circ D_2)(\triangle DEF) = \triangle STU\)

4. \[ \triangle ABC \cong \triangle DEF; \text{ Answers may vary.} \]
   Sample: \((r_{(90^\circ, O)} \circ R_{x-axis})(\triangle ABC) = \triangle DEF\)

5. The solid-line figure is a dilation of the dashed-line figure with center of dilation \(D\). Is the dilation an enlargement or a reduction? What is the scale factor of the dilation? Reduction; \(\frac{1}{4}\)

6. Given \(P(2, 5)\), what are the coordinates of \(D_5(P)\)? \((10, 25)\)

7. Given \(P(10, -20)\), what are the coordinates of \(D_{0.2}(P)\)? \((2, -4)\)

Do you UNDERSTAND?

8. Compare and Contrast  How is a similarity transformation different from an congruence transformation? How are they similar?
   A similarity transformation maps a figure to a similar but not congruent figure. A congruence transformation maps a figure to a congruent figure both are compositions of transformations. A similarity transformation is composition of dilations and rigid motions, while a congruence transformation is composition of just rigid motions.

9. Error Analysis  A student uses image and preimage lengths of a reduction to find a scale factor of 1.5. Explain how you know she has made an error.
   The scale factor < 1; she may have divided the longer length by the shorter length.
Do you know HOW?

1. In the figure at the right, \( \triangle A'B'C' \) is a translation image of \( \triangle ABC \). What is a rule for the translation?
\[ T_{< -5, 6>} (x, y) \]

2. Is a rotation a rigid motion? Explain.
Yes; a rotation preserves distance and angle measure, so it is a rigid motion.

Find the coordinates of the vertices of each image.

3. \( T_{< -1, 2>} (MATH) \)
   \[ M'(-3, 4), A'(2, 2), T'(-1, -2), H'(-5, 0) \]
4. \( R_{x-axis} (MATH) \)
   \[ M'(-2, -2), A'(3, 0), T'(0, 4), H'(-4, 2) \]
5. \( r_{(90°, O)} (MATH) \)
   \[ M'(-2, -2), A'(0, 3), T'(4, 0), H'(2, -4) \]
6. \( D_2 (MATH) \)
   \[ M'(-4, 4), A'(6, 0), T'(0, -8), H'(-8, -4) \]
7. \( (R_{y-axis} \circ T_{< 0, 2>}) (MATH) \)
   \[ M'(2, 4), A'(-3, 2), T'(0, -2), H'(4, 0) \]

Write a single transformation rule that has the same effect as each composition of transformations on the given figure.

8. \( T_{< -3, 4>} \circ T_{< 1, -2>} \)
   \[ T_{< -2, 2>} \]
9. \( R_y = -2 \circ R_y = 2 \)
   \[ T_{< 0, -8>} \]
10. \( R_x = 1 \circ R_y = -1 \quad r_{(180°, (1, -1))} \)

Identify the rigid motion that maps the solid-line figure to the dashed-line figure.

11. translation
12. reflection
Write a congruence or similarity statement for the two figures in each coordinate grid. Then write a congruence transformation or similarity transformation that maps one figure to the other.

13. \( \triangle ABC \cong \triangle DFE; \) Answers may vary.
   Sample: \( (r_{90^\circ}, O) \circ R_{y\text{-axis}}(\triangle ABC) = \triangle DFE \)

14. \( \triangle GHI \sim \triangle JKL; \) Answers may vary.
   Sample: \( D_{0.5}(\triangle GHI) = \triangle JKL \)

15. \( \triangle MNP \cong \triangle QRS; \) Answers may vary.
   Sample: \( (T_{-5, 0} \circ r_{270^\circ}, O)(\triangle MNP) = \triangle QRS \)

16. \( \triangle TUV \sim \triangle XYZ; \) Answers may vary.
   Sample: \( (T_{-4, -3} \circ D_2)(\triangle TUV) = \triangle XYZ \)

Find the image of each point for the given dilation.

17. \( A(5, -10); D_{0.4}(A) \)

18. \( B(-1, -2); D_3(B) \)

19. \( C(4, 6); D_{2.5}(C) \)

Do you UNDERSTAND?

20. Coordinate Geometry \( \triangle XYZ \) has vertices at \( X(-2, 3), Y(3, 0), \) and \( Z(2, 2) \). Graph the image of \( \triangle XYZ \) for the translation \( T_{<3, 1>}(\triangle XYZ) \).

21. Reasoning A transformation maps each point \( (x, y) \) to \( (-kx, ky) \). Write this transformation as a composition of a dilation and rigid motion. \( R_{y\text{-axis}} \circ D_k \)

22. Vocabulary What is the image of \( (2, 3) \) after a reflection across the \( y\)-axis? \( (-2, 3) \)
Performance Tasks

Chapter 9

Task 1

A flea is 0.2 cm long. As it walks across the glass surface of an overhead projector, its image on the screen appears to be 4 cm long. What is the scale factor of the dilation? If the flea jumps 10 cm across the glass surface, how far does its image jump across the screen? Explain your reasoning.

20; 200 cm; all distances are magnified 20 times. [4] Student gives correct answers and an accurate explanation. [3] Two out of three correct answers are given; student used the correct strategy and gives an accurate explanation. [2] Student gives answers and an explanation that are largely correct. [1] Student gives answers or an explanation with serious errors. [0] Student makes little or no effort.

Task 2

A composition of transformations used to describe a congruence transformation is not unique. For example, the composition \( r_{(90\degree, O)} \circ T_{<5, 2>} \) maps the point \((x, y)\) to \((-y - 2, x + 5)\), as does the composition \( T_{<-2, 5>} \circ r_{(90\degree, O)} \). Write another example of transformation composed of a translation followed by a 90\degree rotation, and then write an equivalent transformation composed of a 90\degree rotation followed by a translation. Show that each point \((x, y)\) maps to the same point under both transformations.

Check students’ work. [4] Student gives a correct example of two equivalent compositions and shows all work. [3] Student gives a correct example of two equivalent compositions but shows little work. [2] Student gives a correct example of two equivalent compositions but shows no work. [1] Student gives an example of transformations with serious errors. [0] Student makes little or no effort.

Task 3

In the real world, not all transformations are always possible. For example, if you drive a car around town, what types of transformations does the car undergo? What types can it not undergo? Explain.

Answers may vary. Sample: The car can undergo translations and rotations as it moves and turns. But it cannot undergo a dilation, which would change its size, nor a reflection, which would, for example, put the steering wheel on the passenger side. [4] All correct answers (possible: translation, rotation; not possible: reflection, dilation) and an accurate explanation are given. [3] Three out of four correct answers and an accurate explanation are given. [2] Two out of four correct answers and an explanation that is largely correct are given. [1] Student gives answers or an explanation with serious errors. [0] Student makes little or no effort.
Task 4

Draw a quadrilateral in the coordinate grid at the right. Show the figure undergoing the transformation rules you have studied in this chapter. Use different coordinate grids for each transformation. Draw each image, and write the rule you used.

a. translation $T_{<1, 0>}$

b. dilation

c. reflection $R_{y=1}$

d. glide reflection $R_{y=-2} \circ T_{<2, 0>}$

e. rotation $r_{(90^\circ, O)}$

Cumulative Review
Chapters 1–9

Multiple Choice

1. A woman has a piece of wood that is 22 ft long and another that is 13 ft long. She wants to select another piece of wood so that she can put all the pieces together to make a triangular garden bed. How long could the third piece of wood be?  
   C
   A $8 \text{ ft}$
   B $8 \text{ ft } 6 \text{ in.}$
   C $12 \text{ ft}$
   D $36 \text{ ft}$

2. A scientist looks at small pond organisms using a microscope. In his view, one of the organisms is 2.2 cm long when magnified to 100 times its actual size. What is its actual length, in mm?  
   G
   F $0.022 \text{ mm}$
   G $0.22 \text{ mm}$
   H $22 \text{ mm}$
   I $220 \text{ mm}$

3. For what values of $x$ and $y$ are the triangles congruent?  
   A
   A $x = 3, y = 12$
   B $x = 4, y = 1$
   C $x = 12, y = 3$
   D $x = 3, y = 10$

4. The lengths of the sides of a triangle are in the extended ratio $3 : 7 : 9$. The triangle’s perimeter is 228 m. What are the lengths of the sides?  
   H
   F $30, 70, and 90 \text{ m}$
   G $33, 77, and 107 \text{ m}$
   H $36, 84, and 108 \text{ m}$
   I $37, 84, and 111 \text{ m}$

5. A painter leans a ladder up against a house. The ladder is 12 ft long and the base of the ladder is 3.4 ft away from the house. The ladder reaches the base of a window. To the nearest tenth of a foot, how high off the ground is the base of the window?  
   B
   A $8.6 \text{ ft}$
   B $11.5 \text{ ft}$
   C $11.6 \text{ ft}$
   D $12.5 \text{ ft}$

Short Response

6. A game comes with directions to cut out a figure from paper and fold it into a three-dimensional figure. What type of figure will be formed using the net shown?  
   pentagonal pyramid
   [2] correct answer
   [1] some type of pyramid
   [0] incorrect answer

7. Hexagon $HEXAGZ$ has vertices $H(3, 0), E(1, -2), X(-1, -2), A(-3, 0), G(-2, 2)$, and $Z(1, 2)$. What are the coordinates of the vertices of $D_{3.5}(HEXAGZ)$?  
   $H'(10.5, 0), E'(3.5, -7), X'(-3.5, -7), A'(-10.5, 0), G'(-7, 7), Z'(3.5, 7)$
   [2] all six vertices correct
   [1] at least three vertices correct
   [0] two or fewer vertices correct
   [2] correct answer
   [1] correct answer
   [0] correct answer
Cumulative Review (continued)

Chapters 1–9

Short Response

8. In \( \triangle ABC \), \( AB = 30 \), \( AC = 35 \), and \( m\angle A = 72^\circ \). To the nearest tenth, what are the unknown measures of the triangle? \( BC \approx 38.4 \), \( m\angle B \approx 60.1^\circ \), \( m\angle C \approx 48.0^\circ \) [2] all three measures are correct [1] at least \( BC \) is correct [0] no answers correct

9. \( \triangle ABC \) has vertices \( A(-2, 0) \), \( B(0, 4) \), and \( C(1, 2) \). What are the coordinates of the vertices of \( (r_{(90^\circ, O)} \circ R_{x-axis})(\triangle ABC) \)? \( A'(0, -2) \), \( B'(4, 0) \), \( C'(2, 1) \) [2] all three vertices are correct [1] at least one measurement is correct [0] no answers correct

10. Point \( T \) is the centroid of the triangle.
If \( AZ = 12.36 \), what is \( AT \) ? What is \( TZ \)?
\( AT = 8.24 \); \( TZ = 4.12 \) [2] both answers correct [1] one answer correct [0] no answers correct

11. A ramp leading to a stage forms a \( 37^\circ \) angle with the floor. If the ramp is 22 ft long, how high is the stage from the ground? Round to the nearest inch. 159 in. or 13 ft 3 in. [2] correct answer [1] answer rounded incorrectly [0] incorrect answer

12. The coordinates of the endpoints of \( \overline{AB} \) are \( A(15, 6) \) and \( B(3, 9) \). Find the length of \( \overline{AB} \) to the nearest tenth. What is the midpoint of \( \overline{AB} \)? 12.4; (9, 7.5) [2] both answers correct [1] one answer correct [0] no answers correct

Gridded Response

13. The distance from one corner of a square tablecloth to the opposite corner is 102 in. To the nearest foot, what is the side length of the tablecloth?

14. What is the measure of \( \angle x \)?

Answers

13. 6
14. 6 1
Chapter 9 Project Teacher Notes: Frieze Frames

About the Project
Students will identify frieze patterns that were produced using reflections, translations, rotations, and glide reflections. Students will learn how to classify the patterns by their symmetries. They will also design their own patterns.

Introducing the Project
- Have each team draw a nonregular polygon, trace it, and cut it out. Then have the teams use their cutouts to make patterns.
- Have students try to copy a shape by drawing in a mirror.

Activity 1: Investigating
Ask students to describe wallpaper borders or stencils they have seen that have frieze patterns. Discuss how the patterns are repeated. You may want to have students look through department store catalogs, home-decorating magazines, or wallpaper books for examples. Students must be familiar with translations and reflections.

Activity 2: Classifying
Students also may want to classify the patterns from the previous activities.

Activity 3: Designing
Students may want to divide the work by having each group member make a design for one of the types. Then the group can complete the remaining types together.

Finishing the Project
You may wish to plan a project day on which students share their completed projects. Encourage students to share their processes as well as their products.
- Have students review their sketches, classifications, and design needed for the project.
- Ask students to share any insights they gained when completing the project, such as how they identified different transformations.
Chapter 9 Project: Frieze Frames

Beginning the Chapter Project

Ukrainian painted eggs, dollar bills, Native American pottery, Japanese kimonos, automobile tire treads, and African cloths are products of vastly diverse cultures, but all these things have something in common. They contain strips of repeating patterns, called frieze patterns.

In this chapter project, you will explore the underlying relationships among frieze patterns from around the world. You also will produce your own designs. You will see how distinct civilizations—separated by oceans and centuries—are linked by their use of geometry to express themselves and to beautify their world.

Activities

Activity 1: Investigating

A frieze pattern, or strip pattern, is a design that repeats itself along a straight line. Every frieze pattern can be mapped onto itself by a translation. Some also can be mapped onto themselves by other transformations. Decide whether each frieze pattern can be mapped onto itself by a reflection, a rotation, a translation, or a glide reflection.

a. Nigerian design
   reflection and translation

b. Ancient Egyptian ornament
   rotation and translation

c. Arabian design

Activity 2: Classifying

It may surprise you to find out that when you classify frieze patterns by their symmetries, there are only seven different types. Each pattern is identified by a different two-character code: 11, 1g, m1, 12, mg, 1m, or mm. Use the flowchart at the right to classify each frieze pattern below.

a. Caucasian rug design, Kazakh
   mg

b. French, Empire motif
   12
Activity 3: Designing

In previous activities, you investigated and classified frieze patterns from a variety of cultures. Now you can make your own. Use graph paper, dot paper, geometry or drawing software, or cutouts. Make at least one frieze pattern for each of the seven types summarized at the right.

Finishing the Project

Using one or more of the techniques you explored, make your own design. Prepare a frieze pattern display. Include a brief explanation of frieze patterns as well as your original designs for each of the seven types of frieze patterns. Find and classify more examples of frieze patterns from various cultures, clothing, and other places in your home, school, and community.

Reflect and Revise

Ask a classmate to review your project with you. Together, check that the diagrams and explanations are clear and accurate. Revise your work as needed. Consider doing more research.

Extending the Project

Use logical reasoning and what you’ve learned about transformations to explain why there are no more than seven different frieze patterns. List all other possible combinations of symmetries, and show how each can be ruled out.

The Seven Types of Frieze Patterns

<table>
<thead>
<tr>
<th>Types</th>
<th>Symmetries</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>T</td>
<td>P P P P P P</td>
</tr>
<tr>
<td>12</td>
<td>T, H</td>
<td>Z Z Z Z Z Z</td>
</tr>
<tr>
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</tr>
<tr>
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<td>T, G</td>
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<td>I I I I I I I</td>
</tr>
</tbody>
</table>

Key to Symmetries:

T = Translation
H = Half-turn
RV = reflection in vertical line
RH = Reflection in horizontal line
G = Glide reflection
Chapter 9 Project Manager: Frieze Frames

Getting Started
Read about the project. As you work on it, you will need several sheets of graph paper or dot paper. If available, you may use geometry software or drawing software also. Keep all your work for the project in a folder, along with this Project Manager.

Checklist
☐ Activity 1: transformations
☐ Activity 2: seven types of friezes
☐ Activity 3: pattern design
☐ Frieze Pattern display

Suggestions
☐ Review reflections, rotations, and glide reflections before completing this activity.
☐ Run through the flowchart twice to be sure.
☐ Check your work by reviewing it with a friend.
☐ Use a map or time line to add interest to your display.

Scoring Rubric

4   Your information is accurate and complete. Your diagrams and explanations are clear. You use geometric language appropriately and correctly. You have included many frieze patterns exemplifying a wide variety of styles, cultures, media, and geometric properties. Your display is organized and complete.

3   Your information may contain a few minor errors. Your diagrams and explanations are understandable. Most of the geometric language is used appropriately and correctly. You have included a moderate number of frieze patterns of some variety. Your display shows some organization.

2   Much of the information is incorrect. Diagrams and explanations are hard to follow or misleading. Geometric terms are completely lacking, used sparsely, or often misused. The display does not include many examples and shows little if any attempt at organization.

1   Major elements of the project are incomplete or missing.

0   Project not handed in, or work does not follow instructions.

Your Evaluation of Project  Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of Project